KINGDOM OF BAHRAIN

Ministry of Education

مَمْلَكَةُ البَحْرَيْن

وَزَارَةُ التَّرْبِيَةِ وَالتَّعْلِيم

Teacher's Guide

In Financial Mathematics 2

مال316

2030 البحرين BAHRAIN

The Ministry of Education, Kingdom of Bahrain has decided to teach this book in secondary schools

Teacher's Guide in Financial Mathematics 2 (Fin316)

Secondary Education

First Edition

2024-1445

Authoring and Development
A specialized team from the Ministry of Education and specialists from the Kingdom of Bahrain

THE KING OF THE KINGDOM OF BAHRAIN

Content

The vision of the financial mathematics curriculum:

A curriculum that enhances belonging and consolidates the values of citizenship, by highlighting the role of financial and economic institutions, commercial projects, and production in the service of the national economy, and clarifying areas of commercial cooperation between the Arab Gulf states. As well as providing students with finance experiences and skills such as problem-solving, critical thinking, time management, financial ratio analysis and feasibility study projects, and forming future visions about the various financial activities in the Kingdom of Bahrain and ways to enhance and develop them.

Financial Mathematics Curriculum Message:

- Building a conscious understanding of the Kingdom of Bahrain's Economic Vision 2030 and the goals of the economic development
- Focusing on the study of finance and its practices in project management.
- Developing awareness of the importance of Finance and its role in serving the national economy.
- Developing problem-solving, decision-making and crisis-management skills
- Using strategies that develop thinking skills (creative, innovative)
- Promoting the use of comprehensive evaluation methods for developing higher-order thinking skills.
- Promoting the use the employment of information technology in the study of finance.
- Emphasizing on the employment of 21st century skills in education, such as self-learning, leadership, effective communication, digital culture, and others.

Introduction:

Dear teachers,

We are pleased to present the teacher's guide to finance, hoping that it will be a guide for you in teaching the subject, and a supporter in evaluating students, in order to achieve the desired goals of teaching accounting.

The guide includes:

A) Introduction to the financial mathematics Series:

This introduction explains how to build the series scientifically and pedagogically, and highlights the focal points on which the curriculum focuses in this class. The philosophy of the horizontally balanced and vertically interconnected series, and the various teaching methods used in the guide, Assessment types, and their suggested tools, that take into account individual differences between students.

B) An overview of the chapter:

The course is divided into chapters. The teacher's guide begins in each chapter with an overview that includes an outline of the lessons and their objectives, the sources of their teaching, and the proposed time plan for teaching. Each lesson, then it introduces the vertical coherence of the topic of the chapter during class and other classes. He then provides support to the teacher through the chapter start page in the student's book, and how to benefit from them in presenting the topic of the chapter, as well as highlighting the purpose of the leaflets, their function and when to be use. Then it displays a calendar with its different types and various tools a chart of the calendar with its different types and its various tools.

C) Lessons:

This guide presents some suggested activities that take into finance the individual differences between students, and in a variety of ways, and help the teacher in teaching each lesson. After that, the guide presents the lesson with specific steps, which are:

The Focus: Shows the correlation of key skills before, during and after the lesson.

Teaching: Provides suggestions for the teacher on how to teach the lesson, including the discussion questions and the suggested activities, and highlights the accounting content of the lesson topic, as well as suggests additional examples for the teacher.

Training: It includes various exercises according to the different levels of the students to achieve the objectives of the lesson.

Assessment: Provides suggestions for evaluating the lesson. It also includes ideas for the teacher to verify the extent to which students have comprehended the concepts and mastered the skills presented in the lesson. Moreover, the guide presents a mechanism for following up the handouts. In each lesson, the guide also provides answers to the questions and exercises.

D) Evaluation methods

The series provides a variety of methods for evaluating students (diagnostic, formative, and summative), and mechanisms for dealing with students' errors and difficulties.

As we present this guide to our fellow teachers, we hope that it will gain their interest, meet their requirements for teaching this course, and help them fulfill their targets.

The Importance of the Guide

Dear teacher, the importance of this guide is that it:

- guides you to, how to use multiple learning tools and resources.
- provides you with suggestions related to the teacher's usage of technology
- directs you how to apply various teaching strategies.
- provides you with clear procedures for how to carry out the learning activities.
- shows how to address issues that may arise during the lesson.
- directs you to use a variety of evaluation methods (cognitive - skill - emotional).
- directs you to how to link and integrate between the different parts of the an article.
- directs you to how to link and integrate between the material and other materials.
- guides you on how to identify common errors, and how to deal with them.
- includes guiding models for calendar methods.
- includes a list of educational and academic terms that you need while teaching the subject.
- includes a list of references, and some websites; to be used while it in teaching the subject.

Objectives of the guide

This guide aims to help you, dear teacher, in:

- teaching the curriculum effectively to achieve the desired goals within the framework of active learning.
- organizing and managing effectively the multi-level classroom.
- forming an educational environment based on active learning that makes the learner the focus of the educational process and an active participant in the learning process.
- stimulating learners' motivation towards learning and creating a cooperative atmosphere among them.
- using of various educational means such as compound interest and annuities.
- putting the skill and emotional aspects when teaching financial mathematics curricula.
- using a variety of multi-level evaluation methods.

Vertically Connected Financial Mathematics Approach From Grade 11 to Grade 12

This series introduces you to three dimensions of vertical bonding:

1- **Content Design:**

Threading content is an important process that helps your students verify the exact sequence of content and its sequencing from one level to another. This gives you confidence that the content is delivered, reinforced, and assessed in a timely manner, also helps fill in the gaps and avoid unnecessary repetition, enabling you to direct and adapt your teaching to suit your needs.

2- **Teaching Design:**

The strong vertical correlation between the different teachings, methods starting from the first grade, makes it easier for students to move from the primary stage to the intermediate stage and to the secondary stage. Vocabulary, sensory techniques and aids, lesson plan and treatment reduce the factors of difficulty and confusion that some students encounter as they move through the different grades.

3- **Visual Design:**

The series pages have visual designs that are consistent from one grade to the next, helping students to move smoothly from one stage to the other, and to learn and succeed increases when they become familiar with the way they work with them.

The five keys to success

1- **Concept Maps of Previous Experiences:**

The series considers conceptual maps and their development based on students' results in the business culture course.

2- **Balanced deep content:**

The series has been developed to focus on the skills and topics that are faced at each grade level. Students with difficulties, such as solving.

a) Grade 11 (Financial Mathematics 1- FIN, 111):

- Review Numbers and Currency Exchange.
- Salaries and Wages System
- Discounts and Pricing Goods
- Simple Interest

b) Grade 12 (Financial Mathematics 2- FIN, 316):

- Compound Interest.
- Annuities and Loans Amortization
- Capital Budgeting Decision Model
- Breakeven Analysis
- Ratio Analysis

3- **Continuous evaluation:**

This series includes diagnostic, formative, and summative assessments, and remedial and enrichment plans.

4- Treatment plans and diversification of teaching:

The series provides a three-tiered treatment plan:

a) Daily Treatment:

Various alternatives are identified in the teacher's guide for teaching concepts according to different learning styles.

b) Strategic Treatment:

Teachers use remedial tips and support materials.

c) Intensive treatment:

Provides instructional guidance, supportive vocabulary, and remedial plans to help students succeed.

d) professional development:

The series provides many opportunities for the teacher to develop his professional performance, through additional teaching methods, such as video, computer financial mathematics, and vertically interlinked websites from grade 11 to grade 12.

Research Base for Financial Mathematics Software

The continuous research with students, teachers, academics and experts helps build all mathematics programs from the first grade to the twelfth grade on solid foundations.

a) Software Development Research
   - National Standards Assessment
   - Qualitative research for the needs of the labor market
   - Research related to scientific content

b) Formative Research
   - Teaching research base
   - practice exams
   - teacher advisory committees
   - Academic reviewers and advisors

c) Final Research
   - Experimental indicators of program effectiveness
   - longitudinal studies
   - Program quality assessments

Preparing students for university studies and for the labour market:

This series connects what students learn in secondary school with what they are expected to know when they start university.

How can undergraduate studies, be way better arranged for those considering studying at university?

A strong high school curriculum is a good indicator of college readiness, as students who study secondary school financial mathematics books in this series are more prepared for university than those who have not.

The following are some aspects of preparation for the university study that he developed:

Mental Skills

They are necessary skills for learning content at the university level, and include: critical thinking, problem solving and justification, and every day students who study this series have opportunities to develop higher-order thinking skills.

Scientific Content

The secondary school textbooks of this series are consistent with accurate the international standards to suit university education and sustainable development goals in accordance to Bahrain Vision 2030.

General Skills

That include skills such as: reading comprehension, time management, note taking ... etc. This series provides opportunities to develop these skills through accounting reading guidelines, vocabulary links, prediction guides, and more.

What about students who do not plan to go to university?

Accounting in the modern world of technology is no longer restricted to students who attend universities. One of the studies showed that the training programs that a person who wants to get a job is undergoing requires that this person has a certain level of education in preparing daily entries, analyzing accounting operations, preparing final financial statements, adjusting entries, to be successful at ones job , and preparing balance sheets. Until he succeeds in his work.

Program philosophy:

The vertical connection of this series shows a balanced integration of education. This series provides students with a balanced accounting curve through:

- Developing, enhancing and mastering procedural and arithmetic skills.
- Real-life issues.
- The application of accounting in a solution

The sequence of topics in the three accounting books shows the development of the vertical interrelationship of the cognitive understanding and the procedural skills of accounting through the preparation of daily entries, methods of depreciation of fixed assets, methods of evaluating goods, final financial statements and analysis of financial statements.

Continuity of Education

The learning sequence that is described above illustrates the power of matching a desired outcome with success in algebra. This development process avoids gaps or overlaps between grade levels, and ensures that the

concepts and skills of each grade are built on a solid foundation developed in the previous grades. The same direction is used across all tracks, starting from the eleventh to the twelfth grade.

Teaching Process Balance

- concepts
- skills
- solve problems

Problem Solving Strategy

Problem solving strategies help students learn different ways to tackle difficult problems

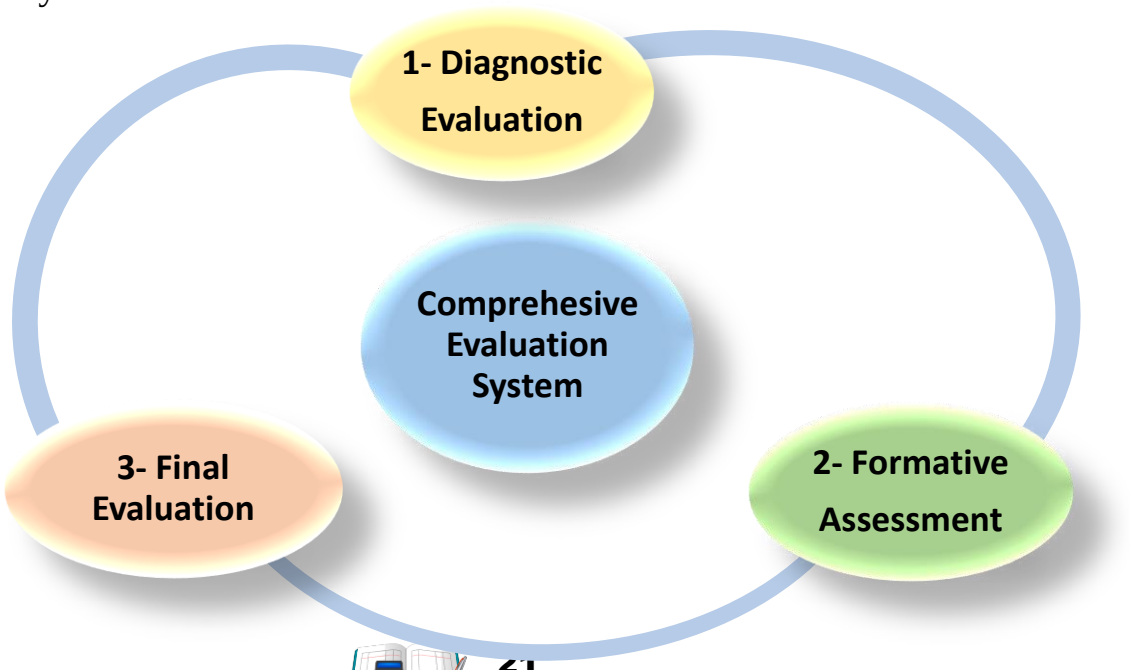
Higher Thinking Skills Problems

These problems require the use of higher-order thinking skills (analysis, composition ...)

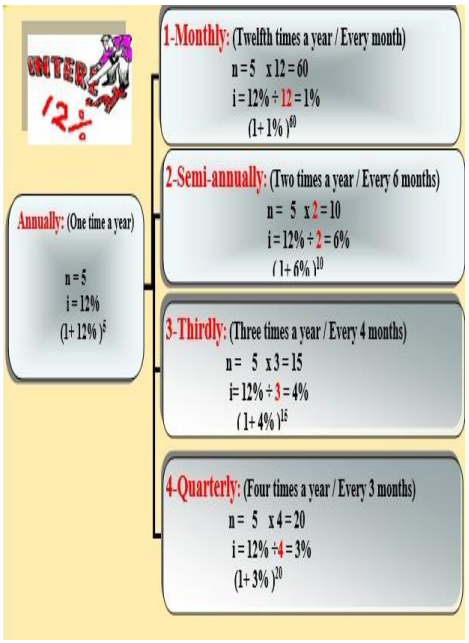
Comprehensive Evaluation System

Error handling

The series provides an on-going, meaningful assessment of the student is progress in the structure of the curriculum and in the supporting materials used by the teacher.



1 Diagnostic Evaluation



a) Preliminary Calendar

Assess your students' knowledge at the beginning of the academic year using diagnostic tests and placement tests. This will help you determine if your students will need additional learning materials and resources; to be able to align with the grade level standards.

b) Academic input level Assessment

Evaluate the knowledge prior to your application at the beginning of the chapter or lesson, using the resources in the Student's Book, the Teacher's Handbook, or any other resources you consider useful.

2 Formative assessment

Formative Assessment

Monitoring Progress: Determine if your students are making adequate progress as they learn each lesson first, using the following types of assessment to vary the teaching and techniques:

Example 1-2-1:

- A trader borrowed BD3000 from a bank for 12 years at 8% annually. Find the future value and compound interest at the end of the period.

Answer:

1- **Future Value:**

$FV = PV \times (1+i)^n$

$= 3000 \times (1+8\%)^{12}$

$= 3000 \times 2.5182 = BD\ 7554.600$

Using FV table for 1BD to find value of $(1+8\%)^{12}$

(by searching under I = 8% and n = 12 periods)

a) Student's Book

- Make sure you understand
- Mid-term exam
- Study and revision guide
- Brochures

b) Teacher's Guide

- Error handling

3 Final Evaluation

Final Evaluation

Evaluate how successful your students are in learning each chapter using the following:

a) Student's Book

- End-of-Semester Exam
- Cumulative Standard Exam
- Research and Reports

b) Teacher's Guide

- Error handling

Activity 1-2-1:

1- Find the value of the following (by using the interest table) :

a - $(1.06)^{12}$ b- $(1.0525)^{60}$

c - $(1.005)^{125}$

2- Bader deposited BD4,200 for 14 years at 5.6% annually. Find the future value at the end of the period.

3- A trader wants to borrow BD20,000 and pays it after 3 years, he has two choices:

- Borrowing on a simple interest at 5¾% annually
- Borrowing on a compound interest at 4% annually

Which choice should he choose? Why?

Diversification of Education

Meet the needs of the students

The series provides broad support that takes into account individual differences among students. Each chapter and each lesson contains suggestions; to determine and meet the needs of your students. The diversification of education meets the needs of the following two groups:

- Below average students.
- Above average students.

Advanced level students

Acceleration and Enrichment: The resources and homework that are rated for A-level students can be used with A-level students.

Multilevel question set

The homework for each lesson was varied according to the levels of the students:

- below average
- within average
- above average

Education Plan

The four-step education plan

Organize your education and include:

1- the focus
2- teaching
3- training
4- Evaluation

The Vertical Correlation at the beginning of each lesson

Outline the objectives that lead to the current lesson content and the objectives that follow, and which come within the scope and sequencer document from grades eleven to twelve.

Reinforcement questions

Each lesson contains some reinforcement questions to be used to help students investigate and understand the main ideas of the lesson.

Additional examples

Each additional example is a reflection of an example in the Student's Book.

Variations of homework

The formative calendar activities provide alternative methods to determine the extent to which students understand at the end of each lesson, such as:

pre-learning

Students connect what they are learning in the current lesson with what they have learned previously.

Later learning

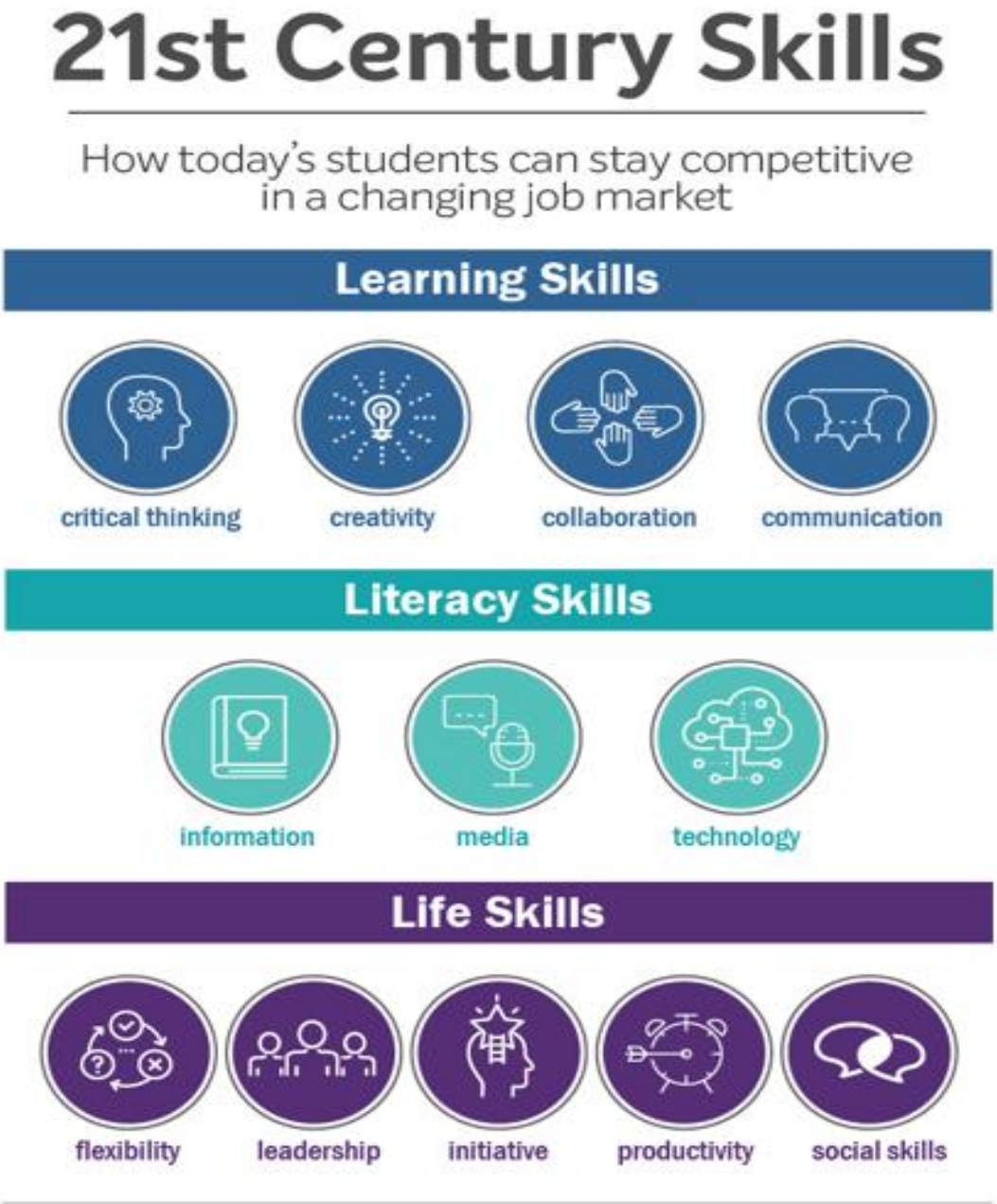
The student anticipates how the current lesson will relate to the next lesson

Nomenclature of Terms in Financial Mathematics

The students specify the financial mathematics information used in the problem.

Exit Card

Write the answer to the question on a sheet of paper and hand over before you leave the classroom.




KINGDOM OF BAHRAIN
Ministry of Education
مملكة البحرين
وزارة التربية والتعليم
FIN 316
Financial
Mathematics (2)
Secondary Level
2030
البحرين
BAHRAIN

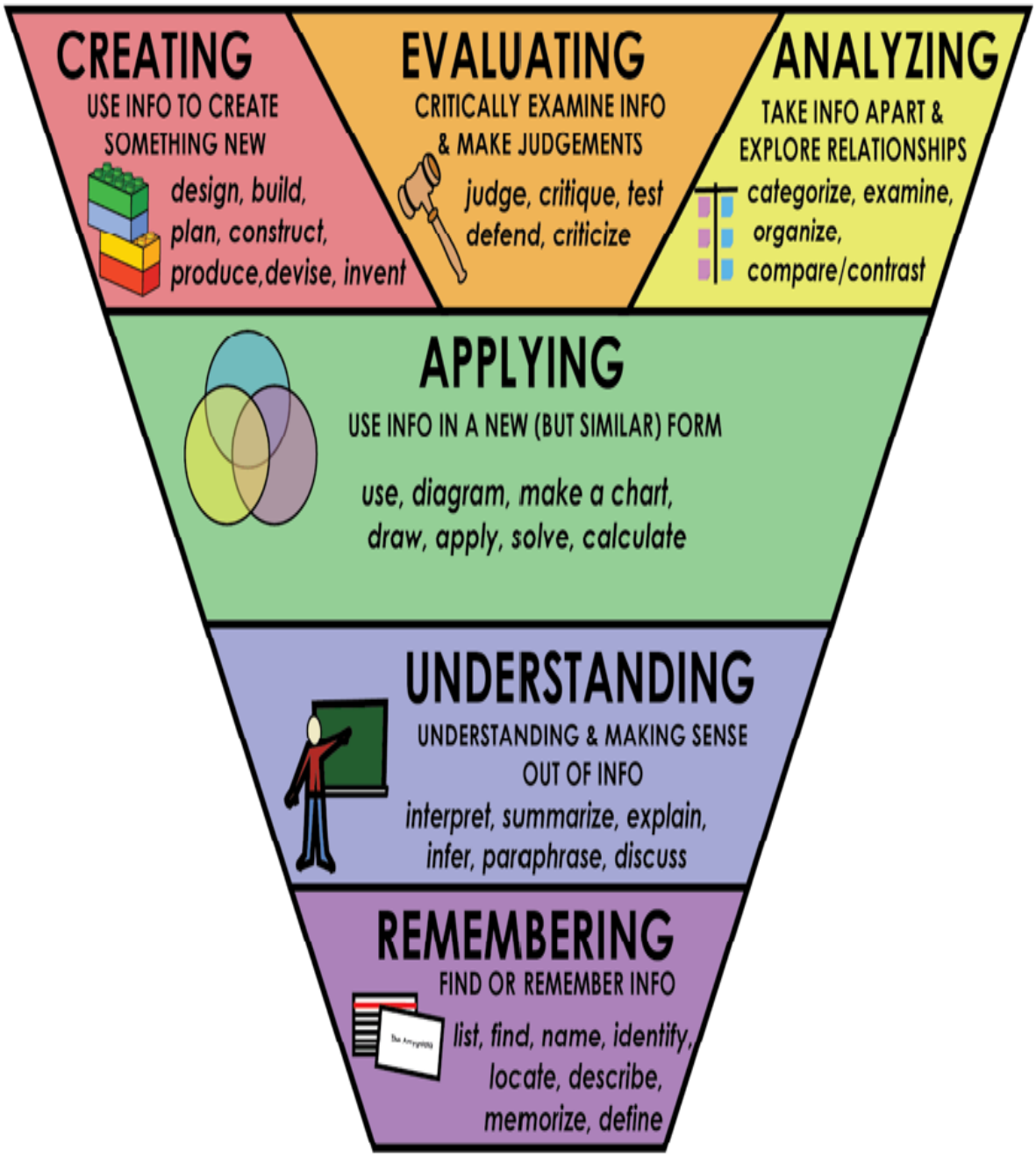
Teaching Plan

	Unit1	Unit2	Unit3	Unit4	Unit5
Title	Compound Interest	Annuities and Amortization Loan	Capital Budgeting Decision Model	Break-even Analysis	Financial Ration Analysis
Studying Period	Three Weeks	Three Weeks	Two Weeks	Two Weeks	Two Weeks
Learning Objectives	1- The difference between SI & CI (using simple interest equation). 2- The calculation of compound interest. 3- The calculation of future and present value. 4- The difference between nominal and partial interest rates. 5- The calculation of CI in more than once a year situation. 6- The calculation of compound interest for changeable interest rate. 7- The calculation of compound interest for changeable investment/loan. 8- The calculation of the factors of compound interest.	1- The concept of annuity/ payments. 2- The different types of annuities. 3- The definition of future and present value of annuities. 4- The calculation of the future and present value of annuities. 5- The calculation of the value of annuity. 6- The preparation of amortized loan schedule	1- The calculation of the payback period. 2- The calculation of the net present value. 3- The calculation of the profitability index.	1- The meaning of break-even point. 2- The calculation of break-even point sales in units. 3- The calculation of break-even point sales in Bahraini Dinar. 4- The calculation of margin of safety in units. 5- The calculation of sales in units to get target profit. 6- The calculation of sales in Bahraini Dinar to get target profit.	1- The definition of the financial ratio. 2- The calculation of and analyze profitability ratios. 3- The benefits of liquidity ratio. 4- The calculation of and analyze liquidity ratio. 5- The benefits of liquidity ratio.

Basic Vocabulary	1- Compound Interest. 2- Future Value. 3- Present Value. 4- Annually 5- Semiannually. 6- Quarterly. 7- Thirdly. 8- Monthly 9- Compounded -Converted	1- Annuity. 2- Ordinary Annuity. 3- Annuity Due. 4- Equal Payments. 5- Amortization Loan.	1- Payback period. 2- Net present value 3- Profitability Index	1- Break-even. 2-Contribution Margin. 3- Magin of Safety. 4- Target of profit	1-Liqiuidity Ratio. 2- Current Ratio. 3- Quick Asid Ratio. 4- Margin of Profit. 5- Capital Employed.
Lesson Resources	1- Text book 2- Teacher's Guide 3- Digital Educational Lesson 4- YouTube Lesson	1- Text book 2- Teacher's Guide 3- Digital Educational Lesson 4- YouTube Lesson	1- Text book 2- Teacher's Guide 3- Digital Educational Lesson 4- YouTube Lesson		1- Text book 2- Teacher's Guide 3- Digital Educational Lesson 4- YouTube Lesson
Teaching Strategies	1- Brainstorming 2- Cooperative Education 3- Problem Solving 4- E-Learning 5- Peer evaluation	1- Brainstorming 2- Cooperative Education 3- Problem Solving 4- E-Learning 5- Peer evaluation	1- Brainstorming 2- Cooperative Education 3- Problem Solving 4- E-Learning 5- Peer evaluation		1- Brainstorming 2- Cooperative Education 3- Problem Solving 4- E-Learning 5- Peer evaluation
Necessary Tools	1- Text book 2- Exchange rate table 3- Calculator	1- Text book 2-Labour law 3- Calculator	1- Text book 2- Follow up on VAT & Tariff 3- Calculator		1- Text book 2- Time table 3- Calculator
Techniques	Smart Interactive Whiteboard	Smart Interactive Whiteboard	Smart Interactive Whiteboard		Smart Interactive Whiteboard
Diversification of Education	Pages (18-23-29-31-33 – 36 -40-41)	Pages (68- 75-76)	Pages (99-100-101)	Pages (116-117-118)	Pages (138-139-140-141

Student assessment methods	1- creative calendar 2- achievement files 3- Practical evaluation 4- self evaluation	1- creative calendar 2- achievement files 3- Practical evaluation 4- self evaluation	1- creative calendar 2- achievement files 3- Practical evaluation 4- self evaluation		1- creative calendar 2- achievement files 3- Practical evaluation 4- self evaluation
Assessment Strategies	1- Performance-based Assessment 2- Pencil and Paper 3- Reflection	1- Performance-based Assessment 2- Pencil and Paper 3- Reflection	1- Performance-based Assessment 2- Pencil and Paper 3- Reflection		1- Performance-based Assessment 2- Pencil and Paper 3- Reflection

BLOOM'S TAXONOMY



Unit 1

Compound Interest

Unit 1: Compound Interest

Explain to students:

1- Calculation the future value.

2- Calculation the present value.

Differentiating between Compound and Simple Interest:

Interest is the cost of borrowing money, where the borrower pays a fee to the lender for the loan. The interest, typically expressed as a percentage, can be either simple or compounded. Simple interest is based on the principal amount of a loan or deposit. In contrast, compound interest is based on the principal amount and the interest that accumulates on it in every period. Simple interest is calculated only on the principal amount of a loan or deposit, so it is easier to determine than compound interest

Lesson Notes

1- The Focus

Before the Lesson: Review simple interest

During the Lesson: 1- The calculation of future and present value. 2- The calculation of CI in more than once a year situation

After the Lesson: The students able to calculate the future and present value.

Calculate Future Value & Interest

Important Points 1-2-1:

Future Value = Present Vale x (1 + interest rate) **Number of periods**

$$FV = PV \times (1+ i)^{n}$$

Tips 1-2-1:

- Where, FV = the future value of the investment at the end of n years.
- PV = the present value, or original amount invested at the beginning of the first year.
- i = the annual interest (or discount) rate.
- n = the number of years during which the compounding occurs.

Example 1-2-1:

► A trader borrowed BD3000 from a bank for 12 years at 8% annually. Find the future value and compound interest at the end of the period.

Answer:

1- Future Value:

$FV = PV \times (1+ i)^n$

$= 3000 \times (1+ 8\%)^{12}$

$= 3000 \times 2.5182 = BD\ 7554.600$

TABLE No.1 Equation: FV = PV × (1 +i)^n									FV for 1BD
N	7.25%	7.5%	7.75%	8%	8.25%	8.5%	8.75%	9%	9.25%
1	1.0725	1.0750	1.0775	1.0800	1.0825	1.0850	1.0875	1.0900	1.0925
2	1.1503	1.1556	1.16101	1.1664	1.1718	1.1772	1.1827	1.1881	1.1936
3	1.2336	1.2423	1.25098	1.2597	1.2685	1.2773	1.2861	1.2950	1.3040
4	1.3231	1.3355	1.34794	1.3605	1.3731	1.3859	1.3987	1.4116	1.4246
5	1.4190	1.4356	1.4524	1.4693	1.4864	1.5037	1.5211	1.5386	1.5563
6	1.5219	1.5433	1.56496	1.5869	1.6090	1.6315	1.6542	1.6771	1.7003
7	1.6322	1.6590	1.68625	1.7138	1.7418	1.7701	1.7989	1.8280	1.8576
8	1.7506	1.7835	1.81693	1.8509	1.8855	1.9206	1.9563	1.9926	2.0294
9	1.8775	1.9172	1.95774	1.9990	2.0410	2.0839	2.1275	2.1719	2.2171
10	2.0136	2.0610	2.10947	2.1589	2.2094	2.2610	2.3136	2.3674	2.4222
11	2.1596	2.2156	2.27295	2.3316	2.3917	2.4532	2.5161	2.5804	2.6463
12	2.3162	2.3818	2.4491	2.5182	2.5890	2.6617	2.7362	2.8127	2.8911
13	2.4841	2.5604	2.63891	2.7196	2.8026	2.8879	2.9756	3.0658	3.1585

Using FV table for 1BD to find value of $(1+8\%)^{12}$

(by searching under I = 8% and n = 12 periods)

2- Compound Interest:

$CI = FV - PV$

$= 7554.600 - 3000$

$= BD\ 4554.600$

Or

$CI = PV \times [(1+i)^n - 1]$

$= 3000 \times [(1+ 8\%)^{12} - 1]$

$= 3000 \times (2.5182 - 1)$

$= 3000 \times 1.5182$

$= BD\ 4554.600$

TABLE No.1 Equation : FV = PV x (1 + i)^n FV for one monetary unit									
n \ i	7.25%	7.5%	7.75%	8%	8.25%	8.5%	8.75%	9%	9.25%
1	1.0725	1.0750	1.0775	1.0800	1.0825	1.0850	1.0875	1.0900	1.0925
2	1.1503	1.1556	1.16101	1.1664	1.1718	1.1772	1.1827	1.1881	1.1936
3	1.2336	1.2423	1.25098	1.2597	1.2685	1.2773	1.2861	1.2950	1.3040
4	1.3231	1.3355	1.34794	1.3605	1.3731	1.3859	1.3987	1.4116	1.4246
5	1.4190	1.4356	1.4524	1.4693	1.4864	1.5037	1.5211	1.5386	1.5563
6	1.5219	1.5433	1.56496	1.5869	1.6090	1.6315	1.6542	1.6771	1.7003
7	1.6322	1.6590	1.68625	1.7138	1.7418	1.7701	1.7989	1.8280	1.8576
8	1.7506	1.7835	1.81693	1.8509	1.8855	1.9206	1.9563	1.9926	2.0294
9	1.8775	1.9172	1.95774	1.9990	2.0410	2.0839	2.1275	2.1719	2.2171
10	2.0136	2.0610	2.10947	2.1589	2.2094	2.2610	2.3136	2.3674	2.4222
11	2.1596	2.2156	2.27295	2.3316	2.3917	2.4532	2.5161	2.5804	2.6463
12	2.3162	2.3818	2.4491	2.5182	2.5890	2.6617	2.7362	2.8127	2.8911
13	2.4841	2.5604	2.63891	2.7196	2.8026	2.8879	2.9756	3.0658	3.1585
14	2.6642	2.7524	2.84343	2.9372	3.0338	3.1334	3.2360	3.3417	3.4506

2- Teaching Reinforcement Question:
Ask the students to read pages 16and 17 (Text Book.)

3- Formative Assessment.
Ask the student to solve example 1-1-2 (page 17) during the class period by using brainstorming Strategy

Study Instructions:
Remember the spelling of numbers from one to thousand.

Activity 1-2-1:

1- Find the value of the following (by using the interest table) :

a - $(1.06)^{12}$ b- $(1.0525)^{60}$ c - $(1.005)^{125}$

2- Bader deposited BD4,200 for 14 years at 5.6% annually. Find the future value at the end of the period.

3- A trader wants to borrow BD20,000 and pays it after 3 years, he has two choices:

- Borrowing on a simple interest at 5¾% annually
- Borrowing on a compound interest at 4% annually

Which choice should he choose? Why?

4- Find the future value of BD1,500 at 9.4% annually for 8 years using a calculator.

5- Find the future value and compound interest for BD2,400 at 4.5% annually for 74 years by using interest tables.

6- A person deposited $6000 for 4 years at 5.5% annually. Find the future value and the compound interest at the end of the period.

Formative assessment:

Ask the students to solve exercises on pages 20, 21 (Textbook.)

➢ **Ask the student to use the following table:**

Chapter Resources			
Resource	**Below Average**	**In Average**	**Over Average**
Teacher's Guide	**Page (17)**	**Page 20 (1-2-1)**	**Page 23 (1-2-2)**
Lesson Resources	**1- Text book 2- Study Guide 3- Digital Educational Lesson 4- YouTube Lesson**	**1- Text book 2- Study Guide 3- Digital Educational Lesson 4- YouTube Lesson**	**1- Text book 2- Study Guide 3- Digital Educational Lesson 4- YouTube Lesson**

Teaching with technology

Ask the student solve the exercises page 33 text book.

Text book Exercises

Activity 1-2-1:

1- Find the value of the following (by using the interest table) :

a - $(1.06)^{12}$ b- $(1.0525)^{60}$ c - $(1.005)^{125}$

Answer:

a - $(1.06)^{12} = 2.0122$

b- $(1.0525)^{60} = (1.0525)^{50} \times (1.0525)^{10}$

$= 12.9153 \times 1.6681 = 21.544$

c - $(1.005)^{125}$ $(1.005)^{50} \times (1.005)^{50} \times (1.005)^{25}$

$= 1.2832 \times 1.2832 \times 1.1328 = 1.8653$

2- Bader deposited BD4,200 for 14 years at 5.6% annually. Find the future value at the end of the period.

Answer:

FV = $PV \times (1+ i)^n$

$= 4200 \times (1+ 5.6\%)^{14}$

$= 4200 \times 2.1443 =$ BD9006.060

3- A trader wants to borrow BD20,000 and pays it after 3 years, he has two choices:

- Borrowing on a simple interest at 5¾% annually
- Borrowing on a compound interest at 4% annually

Which choice should he choose? Why?

Answer:

SI = PV × I × T

= 20000 × 5¾% × 3 = BD3450.

FV = $PV \times (1+ i)^n$

$= 20000 \times (1+ 4\%)^3$

$= 20000 \times 1.1249 =$ BD22498

CI = 22498 -200000 = BD2498

We choose compound interest because its lower than simple interest. But the comparing is not true for difference rate and periods.

4- Find the future value of BD1,500 at 9.4% annually for 8 years using a calculator.

Answer:

$$FV = PV \times (1+i)^n$$

$$= 1500 \times (1+9.4\%)^8$$

$$= 1500 \times 2.0518 = BD\ 3077.700$$

5- Find the future value and compound interest for BD2,400 at 4.5% annually for 74 years by using interest tables.

Answer:

$$FV = PV \times (1+i)^n$$

$$= 2400 \times (1+4.5\%)^{74}$$

$$= 2400 \times (1+4.5\%)^{50} \times (1+4.5\%)^{24}$$

$$= 2400 \times 9.0326 \times 2.8760 = BD\ 62346.618$$

$$CI = FV - PV$$

$$= 62346.618 - 2400 = BD59946.618$$

6- A person deposited $6000 for 4 years at 5.5% annually. Find the future value and the compound interest at the end of the period.

Answer:

$$FV = PV \times (1+i)^n$$

$$= 6000 \times (1+5.5\%)^4$$

$$= 6000 \times 1.2388 = BD\ 7432.800$$

$$CI = FV - PV$$

$$= 7432.800 - 6000 = BD1432.800$$

Activity 1-3-1:

1- Mariam deposited BD2550 in a bank at an effective rate of 6% annually. If the interest is compounded semi-annually. Calculate her fund at the end of 8 years, and then find the compound interest.

Answer:

n = 8 × 2 = 16 times

i = 6 ÷ 2 = 3% semiannually

$FV = PV \times (1+ i)^n$

$= 2550 \times (1+ 3\%)^{16}$

$= 2550 \times 1.6047 = BD\ 4091.985$

$CI = FV - PV$

$= 4091.985 - 2550 = BD1541.985$

2- Faisal wants to borrow KD7200 from a bank to buy a new car if you know the interest rate is 6% annually compounded monthly. Find how much he will pay to the bank at the end of 10 years.

Answer:

n = 10 × 12 = 120 times

i = 6 ÷ 12 = 0.5% monthly

$FV = PV \times (1+ i)^n$

$= 7200 \times (1+ 4.5\%)^{74}$

$= 7200 \times (1.005)^{50} \times (1.005)^{50} \times (1.005)^{20}$

$= 7200 \times 1.2832 \times 1.2832 \times 1.1049$

$= BD\ 13099.182$

3- Sakeena Ahmed deposited BD2600 in a bank at 12% annually compounded quarterly. Find her fund at the end of 5 years and 9 months and find the compound interest.

Answer:

n = 5.75 × 4 = 23 times

i = 12÷ 4 = 3% quarterly

FV = PV × (1+ i) ^{n}

= 2600 × (1+ 3%)^{23}

= 2600 × 1.9736 = BD 5131.360

CI = FV – PV

= 5131.360 – 2600 = BD2531.360

4- A person deposited BD8750 at 3% each quarter – find the future value and the interest at the end of 6 years.

Answer:

n = 6 × 4 = 24 times

i = 3% quarterly

FV = PV × (1+ i) ^{n}

= 8750 × (1+ 3%)^{24}

= 8750 × 2.0328 = BD 17787

CI = FV – PV

= 17787 – 8750 = BD9037

5- A person deposited BD3500 at 4% every 6 months. Find the future value at the end of 8 years and 6 months.

Answer:

n = 8.5 × 2 = 17 times

i = 4% semiannually

FV = PV × (1+ i) ^{n}

= 3500 × (1+ 4%)^{17}

= 3500 × 1.9479 = BD 6817.650

6- Laila borrowed $6400 from a bank at 5% every 4 months. Find the amount she will pay at the end of three years and 8 months. In addition, calculate the interest.

Answer:

$n = (3 + \frac{8}{12}) \times 3 = 11$ times

i = 5% every 4 months

$FV = PV \times (1+ i)^n$

$= 6400 \times (1+ 5\%)^{11}$

$= 6400 \times 1.7103 = BD10945.920$

$CI = FV - PV$

$= 10945.920 - 6400 = BD4545.920$

Activity 1-4-1:

1- Find the future value and compound interest for BD10,000 invested for 4 years, if you know that the rate is changing as follows : 3% annually for the first year, 2.5% annually for the second year, 2% annually for the third year and 1% annually for the fourth year.

Answer:

$FV = PV \times (1+ i)^n$

$= 10000 \times (1+3\%)^1 \times (1+2.5\%)^1 \times (1+2\%)^1 \times (1+1\%)^1$

$= 10000 \times 1.03 \times 1.025 \times 1.02 \times 1.01 = BD\ 10876.337$

$CI = FV - PV$

$CI = 10876.337 - 10000 = BD\ 876.334$

2- What is the future value for BD2,000 invested at 6% annually for 3 years and 4.8% annually for 5 more years?

Answer:

$FV = PV \times (1+ i)^n$

$= 2000 \times (1+6\%)^3 \times (1+4.8\%)^5$

$= 2000 \times 1.1910 \times 1.2642$

= BD 3011.324

3- Find the future value that Saad will get if he saves BD2,000 in a bank for 10 years at changing rate, as following: at 5% annually for the first 5 years, 4.6% annually for the sixth year and 6% annually compounded semi-annually for the rest years.

Answer:

$FV = PV \times (1+i)^n$

$= 2000 \times (1+5\%)^5 \times (1+4.6\%)^1 \times (1+3\%)^8$

$= 2000 \times 1.2763 \times 1.046 \times 1.2668 = BD\ 3382.381$

4- Find the future value for BD2,000 invested for 7 years at 4.5% annually for the first 3 years and 5¾% annually for the rest years.

Answer:

$FV = PV \times (1+i)^n$

$= 2000 \times (1+4.5\%)^3 \times (1+5.75\%)^4$

$= 2000 \times 1.1412 \times 1.2506 = BD\ 2854.369$

Activity 1-4-2:

1- Noora invested BD4000 at Al- Ahli bank at an interest rate of 9% annually and after 3 years she deposited BD3000 to her account at 4% every 4 months. Find the future value and the compound interest at the end of 8 years from the first deposit.

CA for the first 3 years

$FV1 = PV1 \times (1+i)^n$

$= 4000 \times (1+9\%)^3$

$= 4000 \times 1.2950$

$= BD\ 5180$

$CI1 = 5180 - 4000$

$= BD1180$

The balance after deposit

5180

+

3000

= BD8180

CA for the rest 5 years

$FV2 = PV2 \times (1+i)^n$

$= 8180 \times (1+4\%)^{15}$

$= 8180 \times 1.8009$

$= BD\ 14731.362$

$CI2 = 14731.362 - 8180$

$= BD6551.362$

$CI = 1180 + 6551.362 = BD7731.362$

Or $CI = 14731.362 - (4000 + 3000) = BD7731.362$

2- A person deposited BD7,000 at 6% annually, after two years he withdrew BD2,618 from his account and invested the rest at 7.5% annually, Find:

a- His fund after withdrawal

b- The future value for the remaining sum at the end of four years.

CA for the first 2 years

$FV1 = PV1 \times (1+i)^n$

$= 7000 \times (1+6\%)^2$

$= 7000 \times 1.1236$

$= BD\ 7865.200$

The balance after withdrawal

7865.200

−

2618

= 5247.200

CA for the rest 2 years

$FV2 = PV2 \times (1+i)^n$

$= 5247.200 \times (1+7.5\%)^2$

$= 5247.200 \times 1.1556$

$= BD\ 6063.664$

3- Dawood deposited BD3,000 at a compound interest of 8% annually, after 3 years he added BD1220.864 to his account and the rate of interest increased to 10% annually. Find the future value at the end of 8 years from the first deposit.

CA for the first 3 years

$FV1 = PV1 \times (1+i)^n$

$= 3000 \times (1+8\%)^3$

$= 3000 \times 1.2597$

$= BD\ 3779.100$

The balance after deposit

3779.100

+

1220.864

=

BD4999.964

CA for the rest 5 years

$FV2 = PV2 \times (1+i)^n$

$= 4999.964 \times (1+10\%)^5$

$= 8180 \times 1.6105$

$= BD\ 13173.890$

4- A trader borrowed BD20,000 on January 1st, 2000 from BBK and BD15,000 on January 1st,2001 and finally BD30,000 on January 2002. He promised to pay the debt on 31/12/ 2008. If the bank gives compound interest rate of 12% annually, find the amount that should be paid on 31/12/2008.

Answer:

$$FV = PV \times (1+i)^n$$

$$= [20000 \times (1+12\%)^9] + [15000 \times (1+12\%)^8] + [30000 \times (1+12\%)^7]$$

$$= [20000 \times 2.7731] + [15000 \times 2.4760[+ [30000 \times 2.2107]$$

$$= [55462] + [37140] + [66321]$$

$$= BD158923$$

Activity 1-5-1:

1- Find the present value (principal), if the amount after 5 years is BD1469.330 and the compound interest rate is 8% annually, then find the interest.

Answer:

$$PV = FV \div (1+i)^n$$

$$PV = 1469.330 \div (1+8\%)^5$$

$$= 1469.330 \div 1.4693$$

$$= BD\ 1000$$

$$CI = FV - PV$$

$$= 1469.330 - 1000 = BD469.330$$

2- Find the present value (principal), if the amount at the end of 7 years is BD6,450 and the compound interest rate is 4.5% annually for the first three years and 5% annually for the rest years.

Answer:

$$PV = FV \div (1+i)^n$$

$$PV = 6450 \div (1+4.5\%)^3 \div (1+5\%)^4$$

$$= 6450 \div 1.1412 \div 1.2155$$

$$= BD\ 4650$$

3- How much was deposited for an investment of 8% annually compounded quarterly to have an amount of BD2, 228.850 in 5 years?

Answer:

$$n = 5 \times 4 = 20 \text{ times}$$

$$i = 8\% \div 4 = 2\% \text{ quarterly}$$

$$PV = FV \div (1+i)^n$$

$$PV = 2228.850 \div (1+2\%)^{20}$$

$$= 2228.850 \div 1.4859$$

$$= BD\ 1500$$

4- A man deposited money in NBB at 3%annually, after 5 years he withdrew BD2796.500 from his account and invested the rest for 5 years at 2% every 6 months – if the amount at the end of the period is BD3657. Find the present value.

Answer:

$$n = 5 \times 2 = 10 \text{ times}$$

$$i = 2\%$$

1- $$PV2 = FV2 \div (1+i)^n$$

$$PV2 = 3657 \div (1+2\%)^{10}$$

$$= 3657 \div 1.2190$$

$$PV2 = BD\ 3000$$

2- **FV1 = 3000 + 2796.500 = BD5796.500**

3- **PV1 = FV1 ÷ $(1+ i)^n$**

PV1 = 5796.500 ÷ $(1+ 3\%)^5$

= 5796.500 ÷ 1.1593

PV1 = BD 5000

5- Find the present value (principal) that generates an interest of BD700 at 2% annually for 5 years.

Answer:

$$PV = CI \div [(1+ i)^n - 1]$$

PV = 700 ÷ $[(1+ 2\%)^5 - 1]$

= 700 ÷ [1.1041 – 1]

= 700 ÷ 0.1041 = BD 6724.304

6- Taha calculated the compound interest he will get it if he deposits his money in a bank at 3% annually for 20 years and it was BD 4836.667, find the present value.

Answer:

$$PV = CI \div [(1+ i)^n - 1]$$

PV = 4836.667 ÷ $[(1+ 3\%)^{20} - 1]$

= 4836.667 ÷ [1.8061 – 1]

= 4836.667 ÷ 0.8061 = BD 6000

Activity 1-5-2:

1- A loan of BD2,000 amounted to BD3591.713. Find the borrowing. period if the interest rate was 5% annually.

Answer:

$(1+i)^n$ = FV ÷ PV

$(1+5\%)^n = 3591.713 \div 2000$

= 1.7959 we are choosing 5% from table to find n = 12 years

OR $\frac{\log(1.7959)}{\log(1.05)}$ = 12 years

2- Nawal borrowed BD5,000 from a bank at 6% annually. Find the borrowing period if the compound interest was BD 8563.575.

Answer:

FV = CI + PV = 8563.575 +5,000 = BD13563.575

$(1+i)^n$ = FV ÷ PV

$(1+6\%)^n = 13563.575 \div 5000$

= 2.7127 we are choosing 6% from table to find n = 9 years

OR $\frac{\log(2.7127)}{\log(1.06)}$ =17.13 ~ 17 years

3- How long will it take BD2,000 at 2.5% compounded every semi-annually to give an interest of BD 560.169 ?

Answer:

FV = CI + PV

= 560.169 + 2000 = BD2560.169

$(1+i)^n$ = FV ÷ PV

$(1+1.25\%)^n = 2560.169 \div 2000$

= 1.2801 we are choosing 1.25% from table to find n = 20 times = 20 ÷ 2 = 10 Years

OR $\frac{\log(1.2801)}{\log(1.0125)}$ =19.88 ~ 20 times ÷ 2 = 10 Years

4- How long will it take an investment of BD4,000 to amount of BD5,610.400 at 7% annually?

Answer:

$(1+i)^n = FV \div PV$

$(1+7\%)^n = 5610.400 \div 4000$

$= 1.4026$ we are choosing 7% from table to find n = 5 years

OR $\frac{\log(1.4026)}{\log(1.07)} = 5$ years

5- How long will it take the money to double itself at 4% annually?

Answer:

$(1+i)^n = FV \div PV$

$(1+4\%)^n = 2 \div 1$

$= 2$ we are choosing 4% from table to find n = 18 years

OR $\frac{\log(2)}{\log(1.04)} = 17.67 \sim 18$ years

Activity 1-5-3:

1- Saad deposited BD1,400 in a bank that gives compound interest of BD 481.460 at the end of 10 years – Find the interest rate.

Answer:

FV = CI + PV

= 481.460 + 1400 = BD1881.460

$(1+i)^n = FV \div PV$

$(1+i\%)^{10} = 1881.460 \div 1400$

$= 1.3439$ we finding the time by two ways.

I = 3% Annually.

. Press 10^x $\sqrt{}$ 1.3439 = 1.0299 ~ 1.03 – 1 = 0.03 × 100 = 3%

2- Tahera borrowed €4,500 from a bank that gives compound interest of €1,916 at the end of 3 years. Find the quarterly interest rate and the annual rate of interest.

Answer:

FV = CI + PV

= 1916 + 4500 = BD6416

$(1+ i)^n$ = FV ÷ PV

$(1+i\%)^{12}$ = 6416 ÷ 4500 = 1.4258

We finding the time by two ways. From table I = 3% Annually.

Press $^{12x}\sqrt{}$ 1.4258 =1.03– 1 = 0.03 × 100 = 3×4 = 12%

3- If BD 6,600 amounts to BD10,750.740 in 10 years. Find the interest rate.

Answer:

$(1+ i)^n$ = FV ÷ PV

$(1+i\%)^{10}$ = 10750.740 ÷ 6600 = 1.6289

we find the time by two ways. I = 5% Annually.

. Press $^{10x}\sqrt{}$ 1.6289 = 1.05– 1 = 0.05 × 100 = 5%

4- Jawad borrowed BD8,000 for 4 years. If the compound interest was BD 2,948.800 find the semi-annually interest rate, and the annually interest rate.

Answer:

FV = CI + PV

= 2948.800 + 8000 = BD10948.800

$(1+ i)^n$ = FV ÷ PV

$(1+i\%)^{8}$ = 10948.800÷ 8000

= 1.3686 We finding the time by two ways.

I = 4% semiannually × 2 = 8% annually

. Press $\sqrt[8x]{}$ 1.3686 = 1.04 – 1 = 0.04 × 100 = 4% ×2 = 8%

5- Find the interest rate for ¥7,730.325 amounts to ¥10,000 after 13 years.

Answer:

$(1+ i)^n$ = FV ÷ PV

$(1+i\%)^{13}$ = 10000 ÷ 7730.325 = 1.2936

we find the time by two ways. From table I = 2% Annually.

. Press $\sqrt[13x]{}$ 1.2936 = 1.02– 1 = 0.02 × 100 = 2%

Exercises:

1- Find the future value and interest of BD2500 invested in a bank, at the end of 20 years at 5.25% annually.

Answer:

$$FV = PV \times (1+i)^n$$

$$= 2500 \times (1+5.25\%)^{20}$$

$$= 2500 \times 2.7825 = BD\ 6956.250$$

$$CI = FV - PV$$

$$CI = 6956.250 - 2500 = BD\ 4456.250$$

2- Find the future value and compound interest for BD1500 invested at 6.25% annually for 25 years.

Answer:

$$FV = PV \times (1+i)^n$$

$$= 1500 \times (1+6.25\%)^{25}$$

$$= 1500 \times 4.5522 = BD\ 6828.300$$

$$CI = FV - PV$$

$$CI = 6828.300 - 1500 = BD\ 5328.300$$

3- Find the future value for BD8,750 invested for 10 years, if you know that the rate is changing as follows: 5%annually for the first 5 years, 5.25% annually for the sixth year, 5.5% annually for the remaining years.

Answer:

$$FV = PV \times (1+i)^n$$

$$= 8750 \times (1+4.5\%)^{10}$$

$$= 8750 \times (1.05)^5 \times (1.0525)^1 \times (1.055)^4$$

$$= 8750 \times 1.2763 \times 1.0525 \times 1.2388$$

$$= BD\ 14560.763$$

4- Ahmed invested BD7,400 at 6.5% annually, find the future value after 8.5 years.

Answer:

$$FV = PV \times (1+i)^n$$

$$= 7400 \times (1+6.5\%)^{8.5}$$

$$= 7400 \times 1.7079 = BD\ 12638.460$$

5- Moayed deposited BD1400 at 6% annually, find the future value at the end of 10 years and 8 months if you know that interest is compounded quarterly.

Answer:

$n = 10 + (8 \div 12) \times 4 = 42.67$

$I = 6\% \div 4 = 1.5\%$

$$FV = PV \times (1+i)^n$$

$$= 1400 \times (1+1.5\%)^{42.67}$$

$$= 1400 \times 1.8876 = BD\ 2642.64$$

6- Shahd invested BD6000 at 10% annually compounded semi- annually . Find the future value at the end of 4 years and 5 months.

Answer:

$n = 4 + (5 \div 12) \times 2 = 8.83$

$I = 10\% \div 2 = 5\%$

$$FV = PV \times (1+i)^n$$

$$= 6000 \times (1+5\%)^{8.83}$$

$$= 6000 \times 1.5385 = BD\ 9231$$

7- Malak invested BD3150 at 2¼% every semi – annual, find the future value at the end of 4 years and 7 months.

Answer:

n = 4+ (7÷12) × 2 = 9.17

I = 2.25% semi – annual

FV = PV × (1+ i) ^{n}

= 3150 × (1+2.25%)$^{9.17}$

= 3150 × 1.2263 = BD3862.845

8- Omar borrowed BD6,000 from BBK for 5 years at 4%annually compounded semi-annually. Two years including the borrowing date, he borrowed BD2,980.600 at 2% annually compounded quarterly. How much would he will pay after 5 years from first borrowed date?

Answer:

CA for the first 2 years

FV1 = PV1 × (1+ i)^{n}

= 6000 × (1+ 4%)^{2}

= 6000 × 1.0816

= BD 6489.600

The balance after deposit

6489.600

+

2980.600

=

BD9470.200

CA for the rest 5 years

n = 3 × 4 = 12

I = 2% ÷ 4 = 0.5%

FV2 = PV2 × (1+ i)^{n}

= 9470.200 × (1+ 0.5%)^{12}

= 9470.200 × 1.0617

= BD10054.511

9- How long will it take the money to triple itself at 6.75% annually?

Answer:

(1+ i)^{n} = FV ÷ PV

(1+6.75%)^{n} = 3 ÷ 1

= 3 we are choosing 4% from table to find n = 17 years

OR $\frac{\log(3)}{\log(1.0675)}$ **= 16.82 ~ 17 years**

10- Jassim deposited money in BBK for investment at 5% annually, after 5 years he added BD 723.718 to his account at a new interest rate of 5.5% annually. After 3 years, his account was BD2153.781. Find the first sum deposited.

Answer:

PV2 = FV2 ÷ (1+ i)^n

PV2 = 2153.781 ÷ (1+5.5%)^3

= 2153.781 ÷ 1.1742

PV2 = BD 1834.254

1- FV1 = 1834.254 – 723.718 = BD1110.536

2- PV1 = FV1 ÷ (1+ i)^n

PV1 = 1110.536 ÷ (1+ 5%)^5

= 1110.536 ÷ 1.2763

PV1 = BD 870.121

Unit 2
Annuities & Amortization Loan

Learning Objective 2

By the end of this unit, the student should be able to:

- The concept of annuity/ payments
- The different types of annuities
- The definition of future and present value of annuities.
- The calculation of the future and present value of annuities.
- The calculation of the value of annuity.
- The preparation of amortized loan schedule

Unit 2: Annuities & Amortization Loan

Lesson Notes

The teacher explains to student:

1- Difference between ordinary and Annuities due.
2- Calculate FV & PV of ordinary annuities.

2-1-1 Payment (Ordinary)

- Is a payment paid at the end of each period (end of each month– end of each six months – end of each year).

2-2-1 Investment (Due) Annuities.

- Is paid at the beginning of each period (begins every month – begins every six months – begins every year).

Title	Method	Ordinary
Future Value	Table	$FV_n = PMT \times FVIF\ n,i$
	Calculator	$FV_n = PMT \times [\frac{(1+i)^n-1}{i}]$
Present Value	Table	$PV_n = PMT \times PVIF\ n,i$
	Calculator	$PV_n = PMT \times [\frac{1-(1+i)^{-n}}{i}]$

2-3-1

- FV_n : Future value of ordinary annuity.
- PV_n : Present value of ordinary annuity.
- PMT : Equal Payments.
- n : Number of Annuities.
- i : Interest Rate.
- $FVIF\ n,i$: Searching from future value of ordinary annuity table.
- $PVIF\ n,i$: Searching from present value of ordinary annuity table.

The Focus

Before the Lesson:
Revision of compound interest

During the Lesson:

1- The concept of annuity/ payments.

2- The definition of future and present value of ordinary annuities.

3-The calculation of the future and present value of annuities.

After the Lesson:

The student able to count FV and PV of annuities.

Example 2-3-1:

A trader paid an annuity of BD150 at the end of each year for 3 years at an interest rate of 5% annually. Find the following:

1- Future value (amount) and interest at the end of the period.

2- Present value (amount) at the end of the period.

Answer:

$$1-\ FV_n = PMT \times \left[\frac{(1+i)^n-1}{i}\right]$$

$$= 150 \times \left[\frac{(1+5\%)^3-1}{5\%}\right]$$

$$= 150 \times \left[\frac{0.157625}{0.05}\right]$$

$$= 150 \times [3.1525] = \text{BD } 472.875$$

OR by using interest table $= 150 \times 3.1525 = \text{BD } 472.875$

TABLE (FV of Ordinary Annuity) (annuity in arrears … end of period)

n \ i	4.00%	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	11.00%	12.00%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.04000	2.05000	2.06000	2.07000	2.08000	2.09000	2.10000	2.11000	2.12000
3	3.12160	3.15250	3.18360	3.21490	3.24640	3.27810	3.31000	3.34210	3.37440

Important Points 2-3-1: Calculate Compound

$$CI = FV_n - (PMT \times n)$$

$$= 472.875 - (150 \times 3)$$

$$= 472.875 - 450$$

$$= \text{BD } 22.875$$

2- Teaching Reinforcement Question:
Ask the students to study pages 49 to 56 (Text book.)

3- Formative Assessment.
Ask the student to solve the exercise 3 (page 74)
During the class period by using problem solving strategy.

Study Instructions:
Read text book definition FV & PV.

Example 2-3-1:

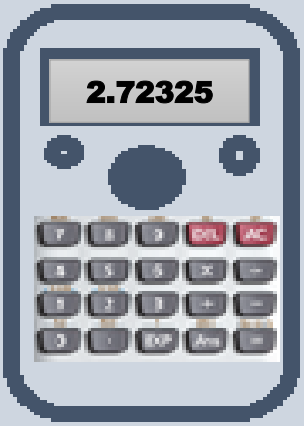
Answer:

2- $PV_n = PMT \times \left[\frac{1-(1+i)^{-n}}{i}\right]$

$= 150 \times \left[\frac{1-(1+5\%)^{-3}}{5\%}\right]$

$= 150 \times \left[\frac{0.1362}{0.05}\right]$

$= 150 \times 2.72325 = BD\ 408.488$



OR by using interest table $= 150 \times 2.72325 = BD\ 408.488$

Teaching with technology

Ask the student to solve the exercise 1 page 74.

Teaching Instruction

Explain to the students example (2-3-) Page 58-59

➢ **Ask the students to use the following table:**

Chapter Resources			
Resource	**Below Average**	**In Average**	**Over Average**
Teacher's Guide	**Page 67 (3)**	**Page 67 (2)**	**Page 67(3)**
Lesson Resources	**1- Text book 2- Study Guide 3- Digital Educational Lesson 4- YouTube Lesson**	**1- Text book 2- Study Guide 3- Digital Educational Lesson 4- YouTube Lesson**	**1- Text book 2- Study Guide 3- Digital Educational Lesson 4- YouTube Lesson**

Formative assessment:

Ask the students to solve exercises 2page 74 (Text Book.)

Text book Exercises

Exercises: page 68

1- Ahmed paid an annuity of BD400 at the end of each year for 7 years at an interest rate of 3% annually. Find the following:

a- Future value (amount) and interest at the end of the period.

b- Present value (amount) at the end of the period.

Answer:

a- $FVn = PMT \times [\frac{(1+i)^n - 1}{i}]$

$FVn = 400 \times [\frac{(1+3\%)^7 - 1}{3\%}]$

$FVn = 400 \times 7.66246 = BD3064.984$

CI = 3064.984 – (400× 7) = BD264.984

b- $PVn = PMT \times [\frac{1-(1+i)^{-n}}{i}]$

$PVn = 400 \times [\frac{1-(1+3\%)^{-7}}{3\%}]$

$PVn = 400 \times 6.23028 = BD2492.112$

2- A trader paid an annuity of BD600 at the beginning of each three months at an interest rate of 6% annually. Find the following:

a- Future value (amount) and interest at the end of 10 years.

b- Present value of the annuities at the end of the period

Answer:

each three months = 4 times a year

No. of annuities (n) = 10 × 4= 40

Partial rate (i) =6÷4= 1.5%

a-$FVnd = 600 \times [\frac{(1+1.5\%)^{40} - 1}{1.5\%}] \times (1 + 1.5\%)$

$FVnd = 600 \times 55.08191 = BD\ 33049.146$

b- $PVn = 600 \times [\frac{1-(1+1.5\%)^{-40}}{1.5\%}] \times (1 + 1.5\%)$

$PVn = 600 \times 29.91585 \times 1.015 = BD18218.753$

OR by using interest table

$PVn = 600 \times 30.36458 = BD\ 18218.748$

3- Calculate the future value and interest of an ordinary annuity of BD800 paid 4 times a year for 6 years if the nominal rate is 4% annually.

Answer:

No. of annuities (n) = 6 × 4 = 24

Partial rate (i) = 4 ÷ 4 = 1%

$$FVn = 800 \times [\frac{(1+1\%)^{24}-1}{1\%}]$$

$$FVn = 800 \times 26.97346 = BD21578.768$$

CI = 21578.768 – (800 × 24) = BD2378.768

4- At the beginning of every 4 months, Nasser deposited an annuity in a bank for 7 years at 9% annually. If the accumulated fund for him became $ 3249.048. Find how much Nasser deposited every 4 months.

Answer:

No. of annuities (n) = 7 × 3 = 21

Partial rate (i) = 9 ÷ 3 = 3%

$$PMT = 3249.048 \div \left[\frac{(1+3\%)^{21}-1}{3\%}\right] \div (1+3\%)$$

$$PMT = 3249.048 \div 28.67649 \div 1.03 = \$\ 110$$

5- Rayan deposited at the middle and at the end of each year an equal payment for 10 years at 5% annually. If the total amount of annuities at the end of the period was BD5236.664. Find the value of each annuity.

Answer:

No. of annuities (n) = 10 × 2= 20

Partial rate (i) =5÷2= 2.5%

at the middle and the end of year n=10×2=20 & i=5÷2=2.5%

$$PMT = 5236.664 \div [\frac{(1+2.5\%)^{20}-1}{2.5\%}]$$

$$PMT = 5236.664 \div 25.54466 = \text{BD } 205$$

6- What semi – annually payment will accumulate to BD1080.549 in five years at 8% annually compounded semi- annually?

Answer:

No. of annuities (n) = 5 × 2= 10

Partial rate (i) =8÷2= 4%

$$PMT = 1080.549 \div [\frac{(1+4\%)^{10}-1}{4\%}]$$

$$PMT = 1080.549 \div 12.00611$$

= BD 90

7- Bilal paid an annuity at the end of every year at 6% annually if the amount of annuities after 10 years was BD922.656. Find the value of each annuity.

$$PMT = 922.656 \div [\frac{(1+6\%)^{10}-1}{6\%}]$$

$$PM = 922.656 \div 13.18079$$

= BD 70.

Exercises: page 75

1- Salman paid an annuity of BD250 at the end of each three months for 9 years at an interest rate of 4% quarterly. Find the following:

a- Future value (amount) and interest at the end of the period.

b- Present value (amount) at the end of the period.

Answer:

a- $FVnd = 250 \times [\frac{(1+4\%)^{36}-1}{4\%}]$

$FVnd = 250 \times 77.59831$= BD 19399.578

b-$PVn = 250 \times [\frac{1-(1+4\%)^{-36}}{4\%}]$

$PVn = 250 \times 18.90828 =$ BD 4727.070

2- A trader paid an annuity of BD900 at the beginning of each six months at an interest rate of 5% annually compounded semiannually.

Find the following:

a- Future value (amount) and interest at the end of 8 years.

b- Present value of the annuities at the end of the period.

Answer:

No. of annuities (n) = 8 × 2= 16

Partial rate (i) =5÷2= 2.5%

a) $FVnd = 900 \times [\frac{(1+2.5\%)^{16}-1}{2.5\%}] \times (1 + 2.5\%)$

$FVnd = 900 \times 19.38022 \times 1.025 = BD\ 17878.253$

OR by using interest table

$FVnd = 900 \times 19.86473 = BD17878.257$

b) $PVn = 900 \times [\frac{1-(1+2.5\%)^{-16}}{2.5\%}] \times (1 + 2.5\%)$

$PVn = 900 \times 13.055 \times 1.025 = BD\ 12043.238$

OR by using interest table

$PVn = 900 \times 13.38138 = BD\ 12043.242$

3- Calculate the future value and interest of an annuity due of BD800 paid 6 times a year for 5 years if the nominal rate is 3.5% annually.

Answer:

No. of annuities (n) = 5 × 6= 30

Partial rate (i) =3.5÷6= 0.58%

$FVnd = 800 \times [\frac{(1+0.58\%)^{30}-1}{0.58\%}] \times (1 + 0.58\%)$

$FVnd = 800 \times 32.66509 \times 1.0058 = BD\ 26283.638$

4- Sajeda deposits a sum of money at the beginning of each year at 4% annually and the amount of annuity became BD1872.96 after 10 years. What was the value of each annuity?

Answer:

$$PMT = 1872.960 \div [\frac{(1+4\%)^{10}-1}{4\%}] \div (1+4\%)$$

$$PMT = 1872.960 \div 12.00611 \div 1.04 = BD150$$

OR by using interest table

$$PMT = 1872.960 \div 12.48635 = BD150$$

5- Awatef paid at the end of each year sum of money at 2% annually so that the amount of annuities after 11 years was BD973.496. What was the value of each annuity?

Answer:

$$PMT = 973.496 \div [\frac{(1+2\%)^{11}-1}{2\%}]$$

$$PMT = 973.496 \div 12.16872 = BD80$$

6- Qassim deposits an equal annuity at the beginning of each year in his fund. The compound interest rate is 2.5% per annum and his balance at the end of 10 years was BD746.428. Find the value of each annuity.

Answer:

$$PMT = 746.428 \div [\frac{(1+2.5\%)^{10}-1}{2.5\%}] \div (1+2.5\%)$$

$$PMT = 746.428 \div 11.20338 \div 1.025 = BD65$$

OR by using interest table

$$PMT = 746.428 \div 11.48347 = BD65$$

7- Abdulla took a loan of BD20,000 from a bank by compound interest rate 12% annually for 8 years, compounded interest every 4 months.

Answer:

No. of annuities (n) = 8 × 3= 24

Partial rate (i) =12÷3= 4%

$$PMT = 20000 \div [\frac{1-(1+4\%)^{-24}}{4\%}]$$

$$PMT = 20000 \div 15.24696 = BD1311.737$$

8- Find the thirdly payment and prepare amortization loan schedule for first two years (first six payments).

year	beginning principal	annual payment	interest expense	principal reduction	remaining principal
1	20000.000	1311.737	800.000	511.737	19488.263
2	19488.263	1311.737	779.531	532.206	18956.057
3	18956.057	1311.737	758.242	553.495	18402.562
4	18402.562	1311.737	736.102	575.635	17826.927
5	17826.927	1311.737	713.077	598.660	17228.267
6	17228.267	1311.737	689.131	622.606	16605.661

9- Find the annually payment and prepare amortization loan schedule for four years, if the loan is BD30,000 for 4 years at compound interest rate 5% annually.

Answer:

$$PMT = 30000 \div [\frac{1-(1+5\%)^{-4}}{5\%}]$$

$$PMT = 30000 \div 3.54595 = \text{BD } 8460.356$$

year	beginning principal	annual payment	interest expense	principal reduction	remaining principal
1	30000.000	8460. 356	1500.000	6960.356	23039.644
2	23039.644	8460. 356	1151.982	7308.374	15731.270
3	15731.270	8460. 356	786.564	7673.792	8057.478
4	8057.478	8460. 356	402.878*	8057.478	0
total		33841.424	3841.424	30000	

10- You are required to prepare amortization loan schedule for a BD40,000 loan to be repaid in equal instalments at the end of each of the next three years. The interest rate is 9% annually, compounded interest semiannually.

Answer:

No. of annuities (n) = 3 × 2= 6

Partial rate (i) =9 ÷ 2= 4.5%

$$PMT = 40000 \div [\frac{1-(1+4.5\%)^{-6}}{4.5\%}]$$

$$= 40000 \div 5.15787 = BD\ 7755.139$$

year	beginning principal	annual payment	interest expense	principal reduction	remaining principal
1	40000.000	7755.139	1800.000	5955.139	34044.861
2	34044.861	7755.139	1532.019	6223.120	27821.741
3	27821.741	7755.139	1251.978	6503.161	21318.580
4	21318.580	7755.139	959.336	6795.803	14522.777
5	14522.777	7755.139	653.525	7101.614	7421.163
6	7421.163	7755.139	333.976*	7421.163	0
total		46530.834	6530.834	40000	

11- Sara borrowed BD85,000 from a bank by compound interest rate 6% annually for 5 years. Find the annual payment and prepare amortization loan schedule for first five years.

Answer:

$$.PMT = 85000 \div [\frac{1-(1+6\%)^{-5}}{6\%}]$$

$$PMT = 85000 \div 4.21236 = BD\ 20178.712$$

year	beginning principal	annual payment	interest expense	principal reduction	remaining principal
1	85000.000	20178.712	5100.000	15078.712	69921.288
2	69921.288	20178.712	4195.277	15983.435	53937.853
3	53937.853	20178.712	3236.271	16942.441	36995.412
4	36995.412	20178.712	2219.725	17958.987	19036.425
5	19036.425	20178.712	1142.287*	19036.425	0
total		100893.560	15893.560	85000	

12- Your family is planning to borrow BD110,000 to purchase a new house on 10-years,8% annual payment. What is the annual payment and prepare amortization loan schedule for the first four years?

Answer:

$$PMT = 110000 \div [\frac{1-(1+8\%)^{-10}}{8\%}]$$

$$PMT = 110000 \div 6.71008 = BD\ 16393.247$$

year	beginning principal	annual payment	interest expense	principal reduction	remaining principal
1	110000.000	16393.247	8800.000	7593.247	102406.753
2	102406.753	16393.247	8192.540	8200.707	94206.046
3	94206.046	16393.247	7536.484	8856.763	85349.283
4	85349.283	16393.247	6827.943	9565.304	75783.979
5	75783.979	16393.247	6062.718	10330.529	65453.450
6	65453.450	16393.247	5236.276	11156.971	54296.479
7	54296.479	16393.247	4343.718	12049.529	42246.950
8	42246.950	16393.247	3379.756	13013.491	29233.459
9	29233.459	16393.247	2338.677	14054.570	15178.889
10	15178.889	16393.247	1214.358	15178.889	0
total		163932.470	53932.470	110000	

Unit 3

Capital Budgeting Decision Model

3

Learning Objective

By the end of this unit, the student should be able to:

- The difference between a short-term and long-term decision.
- The calculation of the payback period.
- The calculation of the net present value.

Unit 3: Capital Budgeting Decision Model

The teacher explains to student:

1- The difference between a short-term and long-term decision.

2- The calculation of the payback period.

Short-Term and Long-Term Decisions

What is the difference between a short-term decision and long-term decision?

We can separate short-term decision a long-term decision into three dimensions:

1. Length of effect
2. Cost
3. Degree of information gathering prior to the decision

The longer the effect and the higher the cost associated with a decision, the greater the time and degree allotted to gathering information on choices and the more sophisticated or complex

Example 3-2-1:

□

► Given the cash flow of three projects A, B and C, and using the payback period decision model, which projects do you accept and which project you will choose ?

Cash Flow	Project A	Project B	Project C
Cost (Cash Outflow)	BD10,000	BD10,000	BD10,000
Cash inflow year 1	BD3,500	BD2,000	BD5,500
Cash inflow year 2	BD3,500	BD3,400	BD4,500
Cash inflow year 3	BD3,500	BD4,000	BD4,000
Cash inflow year 4	BD3,500	BD5,000	BD3,000
Cash inflow year 5	BD3,500	BD6,000	BD2,000

Lesson Notes

The Focus

Before the Lesson:
Revision PV and compound interest

During the Lesson:

1-difference between a short-term and long-term decision.

2- he calculation of the payback period.

After the Lesson:

The student able to accepted or reject the project by using payback period..

Example 3-2-1:

□

Answer:

1. Project A: (Fixed Cash Inflow):

Payback Period = $\frac{Cost\ (\ Initial\ Investment)}{Annual\ Cash\ inflow}$

Payback Period = $\frac{10000}{3500}$ = 2.86 years

2. Project B: (Changeable Cash Inflow):

Year	Cash Flow BD	Yet to be recovered BD	Payback Period Year
0	-10,000		
1	2,000	-10,000+2,000 = -8,000	
2	3,400	- 8,000+3,400 = -4,600	
3	4,000	- 4,600+4,000 = -600	= 3+ $\frac{600}{5000}$ = 3.12 years
4	5,000	- 600+5,000 = 0 (recovered)	
5	6,000	Not used in decision	

3. Project C: (Changeable Cash Inflow):

Year	Cash Flow BD	Yet to be recovered BD	Payback Period Year
0	-10,000		
1	5,500	-10,000 + 5,500 = -4,500	
2	4,500	- 4,500 + 4,500 = 0 (recovered)	2 years
3	4,000	Not used in decision	
4	3,000	Not used in decision	
5	2,000	Not used in decision	

2- Teaching Reinforcement Question:
Ask the students to study pages 78-79 (Text book.)

3- Formative Assessment.
Ask the student to solve the exercise (1) page (3-1 a) During the class period by using problem solving strategy.

Study Instructions:
Recalculate example pages 67-68-69 text book.

Tips 3-2-1:

We can choose the project C because it has the lowest payback period. The company has required 2 years recover period is less than projects A and B. Although the payback period method is used widely, it has two significant weaknesses:

1. It ignored all cash flow after the initial cash outflow has been recovered.
2. It ignored the time value of money.

Teaching with technology

Ask the student to solve the exercise (3-5 a) page 70.

Teaching Instruction

Explain to the students example (3-2-1 on pages 89.

➢ **Ask the students to use the following table:**

Chapter Resources			
Resource	**Below Average**	**In Average**	**Over Average**
Teacher's Guide	**Page 99 (Ex.3- 1-a)**	**Page 99 (Ex. 3-2)**	**Page 99 (Ex 3-5. a)**
Lesson Resources	**1- Text book 2- Study Guide 3- Digital Educational Lesson 4- YouTube Lesson**	**1- Text book 2- Study Guide 3- Digital Educational Lesson 4- YouTube Lesson**	**1- Text book 2- Study Guide 3- Digital Educational Lesson 4- YouTube Lesson**

Formative assessment:

Ask the students to solve exercises 3-6-a page 101(Text Book.)

Text book Exercises

Exercises: page 99

Exercises (3-1):

The following three projects" their cash flow and using 8% discount rate.

Cash Flow	Project A BD	Project B BD	Project C BD
Cost (Initial Investment)	**20,000**	**20,000**	**20,000**
Cash flow year 1	6,000	8,000	3,000
Cash flow year 2	6,000	7,500	4,000
Cash flow year 3	6,000	6,000	5,000
Cash flow year 4	6,000	5,000	6,000

REQUIRED:

Which projects do you accept, and which projects do you reject by using:

a- Payback Period Method.

b- Net Present Value Method.

c- Profitability Index Method.

Answer:

a- Payback period methods

Project (A) – fixed cash flow

$$\textbf{Payback Period} = \frac{Cost\ (Initial\ Investment)}{Annual\ Cash\ inflow}$$

$$\text{Payback Period} = \frac{20000}{6000} = 3.33 \text{ years}$$

Project (B) – Changeable cash flow

Years	Cash flow BD	Yet to be recovered BD	Payback period Year
0	-20,000		
1	8,000	-20,000 + 8,000 = -12,000	
2	7,500	-12,000 + 7,500 = -4,500	
3	6,000	-4,500 + 6,000 = 0 (Recovered)	$= 2+ \frac{4500}{6000} = 2.75$ years
4	5,000	Not used in decision	

Project (C) – Changeable cash flow

Years	Cash flow BD	Yet to be recovered BD	Payback period Years
0	-20,000		
1	3,000	-20,000 + 3,000 = -17,000	
2	4,000	-17,000 + 4,000 = -13,000	
3	5,000	-13,000 + 5,000 = -8,000	
4	6,000	-8,000 + 6,000 = -2,000	
Not recovered			

- **I will choose project (B) because has less years need to recover.**

b- Net present value method

- **Project (A) – fixed cash flow**

Year	Cash Flow (CF)	$\times (1+i)^{-n}$	PV of Cash flow
0	-20,000	$\times (1+8\%)^{0} = 1$	-20,000
1	6,000	$\times (1+8\%)^{-1} = 0.9259$	5,555.4
2	6,000	$\times (1+8\%)^{-2} = 0.8573$	5,143.8
3	6,000	$\times (1+8\%)^{-3} = 0.7938$	4,762.8
4	6,000	$\times (1+8\%)^{-4} = 0.7350$	4,410
Net Present Value (NPV)			-128

NPV = (5,555.4 + 5,143.8 + 4,762.8 + 4,410) – 20,000

= 19,872 – 20,000 = -128 < 0 (Reject)

OR (using table)

PV n = 6,000 × 3.31213 = BD 19,872.780

NPV = 19,872.780 – 20,000 = BD -127.22

PRESENT VALUE OF ORDINARY ANNUITY
(annuity in arrears -- end of period payments)

Periods	RATE PER PERIOD												
	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%	4.00%	5.00%	6.00%	7.00%	8.00%
1	0.99751	0.99502	0.99256	0.99010	0.98522	0.98039	0.97561	0.97087	0.96154	0.95238	0.94340	0.93458	0.92593
2	1.99252	1.98510	1.97772	1.97040	1.95588	1.94156	1.92742	1.91347	1.88609	1.85941	1.83339	1.80802	1.78326
3	2.98506	2.97025	2.95556	2.94099	2.91220	2.88388	2.85602	2.82861	2.77509	2.72325	2.67301	2.62432	2.57710
4	3.97512	3.95050	3.92611	3.90197	3.85438	3.80773	3.76197	3.71710	3.62990	3.54595	3.46511	3.38721	3.31213
5	4.96272	4.92587	4.88944	4.85343	4.78264	4.71346	4.64583	4.57971	4.45182	4.32948	4.21236	4.10020	3.99271

➢ **Project (B) – Changeable cash flow**

Year	Cash Flow (CF)	$\times (1+i)^{-n}$	PV of Cash flow
0	-20,000	$\times (1+8\%)^{0} = 1$	-20,000
1	8,000	$\times (1+8\%)^{-1} = 0.9259$	7,407.2
2	7,500	$\times (1+8\%)^{-2} = 0.8573$	6,429.75
3	6,000	$\times (1+8\%)^{-3} = 0.7938$	4,762.8
4	5,000	$\times (1+8\%)^{-4} = 0.7350$	3,675
Net Present Value (NPV)			**2,274.75**

NPV = 2,274.75 > 0 (Acceptable)

➢ **Project (C) – Changeable cash flow**

Year	Cash Flow (CF)	$\times (1+i)^{-n}$	PV of Cash flow
0	-20,000	$\times (1+8\%)^{0} = 1$	-20,000
1	3,000	$\times (1+8\%)^{-1} = 0.9259$	2,777.7
2	4,000	$\times (1+8\%)^{-2} = 0.8573$	3,429.2
3	5,000	$\times (1+8\%)^{-3} = 0.7938$	3,969
4	6,000	$\times (1+8\%)^{-4} = 0.7350$	4,410
Net Present Value (NPV)			**-5414.1**

NPV = -5,414.1 < 0 (Reject)

- **I will choose project (B) because it has the highest NPV.**

c- Profitability index methods

- **Project (A) – fixed cash flow**

$= \frac{20000-128}{20000} = 0.9936 < 1$ (Reject)

- **Project (B) – Changeable cash flow**

$= \frac{20000+2274.75}{20000} = 1.1137375 > 1$ (Accept)

- **Project (C) – Changeable cash flow**

$= \frac{20000-5414.1}{20000} = 0.9936 < 1$ (Reject)

- **I will choose project (B) , because it has the highest PI.**

Exercises (3-2):

What are the payback periods projects R, S and T? Assume all the cash flow is evenly spread throughout the year. If the cutoff period is three years, which projects do you accept?

Cash Flow	Project R BD	Project S BD	Project T BD
Cost (Initial Investment)	**12,000**	**15,000**	**10,000**
Cash flow year 1	5,000	4,000	7,000
Cash flow year 2	8,000	6,500	3,500
Cash flow year 3	9,000	7,000	3,000

Answer:

Project (R)

Years	Cash flow BD	Yet to be recovered BD	Payback period Year
0	-12,000		
1	5,000	-12,000 + 5,000 = -7,000	
2	8,000	-7,000 + 8,000 (Recovered)	$= 1+ \frac{7000}{8000} = 1.875$ years
3	9,000	Not used in decision	

Project (S)

Years	Cash flow BD	Yet to be recovered BD	Payback period Year
0	-15,000		
1	4,000	-15,000 + 4,000 = -11,000	
2	6,500	-11,000 + 6,500 = 4,500	
3	7,000	-4,500 + 7,000 (Recovered)	$= 2+ \frac{4500}{7000} = 2.643$ years

Project (T)

Years	Cash flow BD	Yet to be recovered BD	Payback period Year
0	-10,000		
1	7,000	-10,000 + 7,000 = -3,000	
2	3,500	-3,000 + 3,500 (Recovered)	$= 1+ \frac{3000}{3500} = 1.857$ years
3	3,000	Not used in decision	

I will choose project (T) because it has the lowest payback period.

Exercises (3-3):

Mona Company has three projects, given discount rates and future cash flow of each project. Calculate the net present value of each project and identify which project do you accept?

Cash Flow	Project E BD	Project F BD	Project G BD
Cost (Initial Investment)	**14,000**	**18,000**	**22,000**
Cash flow year 1	3,200	9,100	5,700
Cash flow year 2	4,800	7,450	5,700
Cash flow year 3	5,300	7,000	6,250
Cash flow year 4	6,900	5,800	7,700
Discount rate	6%	7%	8%

Answer:

Project (E)

Year	Cash Flow (CF)	$\times (1+i)^{-n}$	PV of Cash flow
0	-14,000	$\times (1+6\%)^{0} = 1$	-14,000
1	3,200	$\times (1+6\%)^{-1} = 0.9434$	3,018.88
2	4,800	$\times (1+6\%)^{-2} = 0.8900$	4,272
3	5,300	$\times (1+6\%)^{-3} = 0.8396$	4,449.88
4	6,900	$\times (1+6\%)^{-4} = 0.7921$	5,465.49
Net Present Value (NPV)			**3206.25**

NPV = 3,206.25 > 0 (Accept)

Project (F)

Year	Cash Flow (CF)	$\times (1+i)^{-n}$	PV of Cash flow
0	-18,000	$\times (1+7\%)^{0} = 1$	-18,000
1	9,100	$\times (1+7\%)^{-1} = 0.9346$	8,504.86
2	7,450	$\times (1+7\%)^{-2} = 0.8734$	6,506.83
3	7,000	$\times (1+7\%)^{-3} = 0.8163$	5,714.1
4	5,800	$\times (1+7\%)^{-4} = 0.7629$	4,424.82
Net Present Value (NPV)			**7150.61**

NPV = 7,150.61 > 0 (Accept)

Project (G)

Year	Cash Flow (CF)	$\times (1+i)^{-n}$	PV of Cash flow
0	-22,000	$\times (1+8\%)^{0} = 1$	-22,000
1	5,700	$\times (1+8\%)^{-1} = 0.9259$	5,277.63
2	5,700	$\times (1+8\%)^{-2} = 0.8573$	4,886.61
3	6,250	$\times (1+8\%)^{-3} = 0.7938$	4,961.25
4	7,700	$\times (1+8\%)^{-4} = 0.7350$	5,659.5
Net Present Value (NPV)			**-1215.01**

NPV = -1,215.01 < 0 (Reject)

I will choose project (F) has high NPV more than project (E)

Exercises (3-4):

Given the discount rates and future cash flow of each project listed. Use the profitability index method and identify which project do you accept.

Cash Flow	Project A BD	Project B BD	Project C BD
Cost (Initial Investment)	**16,000**	**16,000**	**16,000**
Cash flow year 1	5,000	6,200	3,650
Cash flow year 2	5,000	5,350	3,600
Cash flow year 3	5,000	5,150	4,550
Cash flow year 4	5,000	4,400	4,750
Discount rate	4%	5%	6%

Answer:

Project (A) - Using table

PRESENT VALUE OF ORDINARY ANNUITY
(annuity in arrears -- end of period payments)

Periods	RATE PER PERIOD												
	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%	4.00%	5.00%	6.00%	7.00%	8.00%
1	0.99751	0.99502	0.99256	0.99010	0.98522	0.98039	0.97561	0.97087	0.96154	0.95238	0.94340	0.93458	0.92593
2	1.99252	1.98510	1.97772	1.97040	1.95588	1.94156	1.92742	1.91347	1.88609	1.85941	1.83339	1.80802	1.78326
3	2.98506	2.97025	2.95556	2.94099	2.91220	2.88388	2.85602	2.82861	2.77509	2.72325	2.67301	2.62432	2.57710
4	3.97512	3.95050	3.92611	3.90197	3.85438	3.80773	3.76197	3.71710	3.62990	3.54595	3.46511	3.38721	3.31213
5	4.96272	4.92587	4.88944	4.85343	4.78264	4.71346	4.64583	4.57971	4.45182	4.32948	4.21236	4.10020	3.99271

PV n = 5,000 × 3.6299 = BD 18,149.5

NPV = 18,149.5 – 16,000 = BD 2149.5

$PI = \frac{16000+2149.5}{16000} = 1.1343 > 1$ (Accept)

Project (B)

Year	Cash Flow (CF)	$\times (1+i)^{-n}$	PV of Cash flow
0	-16,000	$\times (1+5\%)^{0} = 1$	-16,000
1	6,200	$\times (1+5\%)^{-1} = 0.9524$	5,904.88
2	5,350	$\times (1+5\%)^{-2} = 0.9070$	4,852.45
3	5,150	$\times (1+5\%)^{-3} = 0.8638$	4,448.57
4	4,400	$\times (1+5\%)^{-4} = 0.8227$	3619.88
Net Present Value (NPV)			**2825.78**

$$PI = \frac{16000+2825.88}{16000} = 1.1766 > 1 \text{ (Accept)}$$

Project (C)

Year	Cash Flow (CF)	$\times (1+i)^{-n}$	PV of Cash flow
0	-16,000	$\times (1+6\%)^{0} = 1$	-16,000
1	3,650	$\times (1+6\%)^{-1} = 0.9434$	3443.41
2	3,600	$\times (1+6\%)^{-2} = 0.8900$	3204
3	4,550	$\times (1+6\%)^{-3} = 0.8396$	3820.18
4	4,750	$\times (1+6\%)^{-4} = 0.7921$	3762.475
Net Present Value (NPV)			**-1769.935**

$$PI = \frac{16000-1769.935}{16000} = 0.8894 < 1 \text{ (Reject)}$$

I will choose project (B) because has higher PI

Exercises (3-5):

Abdulla Company has three potential projects, all with an initial cost of BD 30,000. Given the discount rates and future cash flow of each project. Which project do you accept? By using:

a- Payback Period Method.

b- Net Present Value Method.

c- Profitability Index Method.

Cash Flow	Project R BD	Project S BD	Project T BD
Cash flow year 1	10,000	13,500	7,300
Cash flow year 2	10,000	12,000	8,800
Cash flow year 3	10,000	10,500	9,750
Cash flow year 4	10,000	7,200	11,600
Cash flow year 5	10,000	6,500	14,000
Discount rate	5.50%	5.50%	5.50%

Answer:

a- Payback period methods

Project (R) – fixed cash flow

$$\textbf{Payback Period} = \frac{Cost\ (Initial\ Investment)}{Annual\ Cash\ inflow}$$

$$\text{Payback Period} = \frac{30000}{10000} = 3 \text{ years}$$

Project (S) – Changeable cash flow

Years	Cash flow BD	Yet to be recovered BD	Payback period Year
0	-30,000		
1	13,500	-30,000 + 13,500 = -16,500	
2	12,000	-16,500 + 12,000 = -4,500	
3	10,500	-4,500 + 10,500 = 6000 (Recovered)	$= 2+ \frac{4500}{10500} = 2.43$ years
4	7,200	Not used in decision	
5	6,500	Not used in decision	

Project (T) – Changeable cash flow

Years	Cash flow BD	Yet to be recovered BD	Payback period Years
0	-30,000		
1	7,300	-30,000 + 7,300 = -22,700	
2	8,800	-22,700+ 8,800 = -13,900	
3	9,750	-13,900 + 9,750 = -4,150	
4	11,600	-4,150+ 11,600 = 7450	$= 3+ \frac{4150}{11600} = 3.36$ years
5	14,000	Not used in decision	

- **I will choose project (S) because has less years need to recover.**

b- Net present value method:

- **Project (R) – fixed cash flow**

Year	Cash Flow (CF)	$\times (1+i)^{-n}$	PV of Cash flow
0	-30,000	$\times (1+5.5\%)^{0} = 1$	-30,000
1	10,000	$\times (1+5.5\%)^{-1} = 0.9479$	9479
2	10,000	$\times (1+5.5\%)^{-2} = 0.8985$	8985
3	10,000	$\times (1+5.5\%)^{-3} = 0.8516$	8516
4	10,000	$\times (1+5.5\%)^{-4} = 0.8072$	8072
5	10,000	$\times (1+5.5\%)^{-5} = 0.7651$	7651
Net Present Value (NPV)			**12703**

NPV = 12703 > 0 (Accept)

- **Project (S) – Changeable cash flow**

Year	Cash Flow (CF)	$\times (1+i)^{-n}$	PV of Cash flow
0	-30,000	$\times (1+5.5\%)^{0} = 1$	-30,000
1	13,500	$\times (1+5.5\%)^{-1} = 0.9479$	12796.650
2	12,000	$\times (1+5.5\%)^{-2} = 0.8985$	10782
3	10,500	$\times (1+5.5\%)^{-3} = 0.8516$	8941.800
4	7,200	$\times (1+5.5\%)^{-4} = 0.8072$	5811.840
5	6,500	$\times (1+5.5\%)^{-5} = 0.7651$	4973.150
Net Present Value (NPV)			13305.44

NPV = 13305.44 > 0 (Accept)

➢ **Project (T) – Changeable cash flow**

Year	Cash Flow (CF)	$\times (1+i)^{-n}$	PV of Cash flow
0	-30,000	$\times (1+5.5\%)^{0} = 1$	-30,000
1	7,300	$\times (1+5.5\%)^{-1} = 0.9479$	6919.670
2	8,800	$\times (1+5.5\%)^{-2} = 0.8985$	7906.800
3	9,750	$\times (1+5.5\%)^{-3} = 0.8516$	8303.100
4	11,600	$\times (1+5.5\%)^{-4} = 0.8072$	9363.520
5	14,000	$\times (1+5.5\%)^{-5} = 0.7651$	10711.400
Net Present Value (NPV)			**13204.490**

NPV = 13204.490 > 0 (Accept)

- **I will choose project (S) because it has the highest NPV.**

c- Profitability index method:

➢ **Project (R) – fixed cash flow**

$= \frac{30000+12703}{30000} = 1.4234 > 1$ (Accept)

➢ **Project (S) – Changeable cash flow**

$= \frac{30000+13305.440}{30000} = 1.4435 > 1$ (Accept)

➢ **Project (T) – Changeable cash flow**

$= \frac{30000+13204.490}{30000} = 1.4401 > 1$ (Accept)

- **I will choose project (S), because it has the highest PI.**

Exercises (3-6):

ALAMAL Company has three potential projects, all with an initial cost of BD 75,000. Given the discount rates and future cash flow of each project. Which project do you accept? By using:

a- Payback Period Method.

b- Net Present Value Method.

c- Profitability Index Method.

Cash Flow	Project R BD	Project S BD	Project T BD
Cash flow year 1	30,000	15,500	25,000
Cash flow year 2	28,000	23,000	25,000
Cash flow year 3	24,000	30,500	25,000
Cash flow year 4	21,000	32,200	25,000
Discount rate	7.00%	7.50%	8.00%

Answer:

d- **Payback period methods**

Project (R) – Changeable cash flow

Years	Cash flow BD	Yet to be recovered BD	Payback period Year
0	-75,000		
1	30,000	-75,000 + 30,000 = -45,000	
2	28,000	-45,000 + 28,000 = -17,000	
3	24,000	-17,000 + 24,000 = 7000 (Recovered)	$= 2+ \frac{17000}{24000} = 2.71$ years
4	21,000	Not used in decision	

➢ **Project (S) – Changeable cash flow**

Years	Cash flow BD	Yet to be recovered BD	Payback period Years
0	-75,000		
1	15,500	-75,000 + 15,500 = -59,500	
2	23,000	-59,500 + 23,000 = -36,500	
3	30,500	-36,500 + 30,500 = -6,000	
4	32,200	-6,000 + 32,200 = 26,200	$= 3+ \frac{6000}{32200} = 3.19$ years

➢ **Project (T) – fixed cash flow :**

Payback Period = $\frac{Cost\ (Initial\ Investment)}{Annual\ Cash\ inflow}$

Payback Period = $\frac{75000}{25000}$ = 3 years

- **I will choose project (S) because has less years need to recover.**

b- Net present value method

➢ **Project (R) – Changeable cash flow**

Year	Cash Flow (CF)	$\times (1+i)^{-n}$	PV of Cash flow
0	-75,000	$\times (1+7\%)^{0} = 1$	-75,000
1	30,000	$\times (1+7\%)^{-1} = 0.9346$	28038
2	28,000	$\times (1+7\%)^{-2} = 0.8734$	24455.200
3	24,000	$\times (1+7\%)^{-3} = 0.8163$	19591.200
4	21,000	$\times (1+7\%)^{-4} = 0.7629$	16020.900
Net Present Value (NPV)			**13105.300**

NPV = 13105.300 > 0 (Accept)

➢ **Project (S) – Changeable cash flow**

Year	Cash Flow (CF)	$\times (1+i)^{-n}$	PV of Cash flow
0	-75,000	$\times (1+7.5\%)^{0} = 1$	-75,000
1	15,500	$\times (1+7.5\%)^{-1} = 0.9302$	14418.100
2	23,000	$\times (1+7.5\%)^{-2} = 0.8653$	19901.900
3	30,500	$\times (1+7.5\%)^{-3} = 0.8050$	24552.500
4	32,200	$\times (1+7.5\%)^{-4} = 0.7488$	24111.360
Net Present Value (NPV)			**7983.86**

NPV= 7983.86 > 0 (Accept)

➢ **Project (T) – fixed cash flow**

Year	Cash Flow (CF)	$\times (1+i)^{-n}$	PV of Cash flow
0	-20,000	$\times (1+8\%)^{0} = 1$	-75,000
1	25,000	$\times (1+8\%)^{-1} = 0.9259$	23147.500
2	25,000	$\times (1+8\%)^{-2} = 0.8573$	21432.500
3	25,000	$\times (1+8\%)^{-3} = 0.7938$	19845
4	25,000	$\times (1+8\%)^{-4} = 0.7350$	18375
Net Present Value (NPV)			**7800**

NPV= 7800 > 0 (Accept)

OR (using table)

PV n = 25,000 × 3.31213 = BD82803.250

NPV = 82803.250– 75,000 = BD 7803.25

PRESENT VALUE OF ORDINARY ANNUITY
(annuity in arrears -- end of period payments)

Periods	RATE PER PERIOD												
	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%	4.00%	5.00%	6.00%	7.00%	8.00%
1	0.99751	0.99502	0.99256	0.99010	0.98522	0.98039	0.97561	0.97087	0.96154	0.95238	0.94340	0.93458	0.92593
2	1.99252	1.98510	1.97772	1.97040	1.95588	1.94156	1.92742	1.91347	1.88609	1.85941	1.83339	1.80802	1.78326
3	2.98506	2.97025	2.95556	2.94099	2.91220	2.88388	2.85602	2.82861	2.77509	2.72325	2.67301	2.62432	2.57710
4	3.97512	3.95050	3.92611	3.90197	3.85438	3.80773	3.76197	3.71710	3.62990	3.54595	3.46511	3.38721	3.31213
5	4.96272	4.92587	4.88944	4.85343	4.78264	4.71346	4.64583	4.57971	4.45182	4.32948	4.21236	4.10020	3.99271

- **I will choose project (R) because it has the highest NPV.**

c- Profitability index methods

- **Project (R) – fixed cash flow**

$= \frac{75000+13105.300}{75000} = 1.1747 > 1$ (Accept)

- **Project (S) – Changeable cash flow**

$= \frac{75000+7983.86}{75000} = 1.1065 > 1$ (Accept)

- **Project (T) – Changeable cash flow**

$= \frac{75000+7800}{75000} = 1.104 > 1$ (Accept)

- **I will choose project (R), because it has the highest PI.**

Unit 4

Break-even Analysis

4

Learning Objective

By the end of this unit, the student should be able to:

- The meaning of break-even point.
- The calculation of break-even point sales in units.
- The calculation of break-even point sales in Bahraini Dinar.
- The calculation of margin of safety in units.
- The calculation of sales in units to get target profit.
- The calculation of sales in Bahraini Dinar to get target

Unit 4: Break-even Analysis

Lesson Notes

The teacher explains to student:

1- The meaning of break-even point.

2- The calculation of break-even point sales in units.

3- The calculation of break-even point sales in Bahraini Dinar.

Objective and uses of break-even

The objective of break-even analysis is to determine the break-even quantity of output by studying the relationships among the firm's cost structure, volume of output, and profit. Alternatively, the firm ascertains the break-even level of sales in Bahraini Dinar that corresponds to the break-even quantity of output. We will develop the fundamental relationships by concentrating on units of output, and then extend the procedure to permit direct calculation of the break-even sales level.

The Focus

Before the Lesson:
Discussion about break-even point.

During the Lesson:

1- calculation of break-even point sales in units

2- calculation of break-even point sales in Bahraini Dinar.

After the Lesson:

The student able to count break-even in units and sales BD.

Important Points 4-2-1: Formulas of break-even

1- Unit Contribution Margin = Selling price per unit - Variable cost per unit

$$UCM = USP - UVC$$

2- Contribution Margin Percentage $= \frac{\text{Selling price per unit} - \text{Variable cost per unit}}{\text{Selling price per unit}} \times 100$

$$CM\% = \frac{USP-UVC}{USP} \times 100$$

3- Break-even point sales in units $= \frac{\text{Fixed Cost}}{\text{Selling price per unit} - \text{Variable cost per unit}}$

$$Unit\ Sales\ /_{BEP} = \frac{FC}{USP-UVC}$$

4- Break-even point sales in BD $= \frac{\text{Fixed Cost}}{\text{Contribution Margin Percentage}}$

$$Sales\ in\ BD\ /_{BEP} = \frac{FC}{CM\%}$$

OR

Break-even point in sales BD = Break-even point in sales units × Unit Selling Price

$$Sales\ in\ BD\ /_{BEP} = Unit\ Sales\ /_{BEP} \times USP$$

Example 4-2-1:

ABC Company sold a computer at BD20 per unit and it had variable cost of BD12 per unit. The total annual fixed cost is BD160,000.

Required:

1) Calculate contribution margin per unit.
2) Calculate contribution margin percentage.
3) Calculate break-even point sales in units.
4) Calculate break-even point sales revenue in BD.

Answer:

1- Unit Contribution Margin = Selling price per unit - Variable cost per unit

$$UCM = USP - UVC$$
$$= 20 - 12 = BD8$$

2- Contribution Margin Percentage $= \frac{\text{Selling price per unit} - \text{Variable cost per unit}}{\text{Selling price per unit}} \times 100$

$$CM\% = \frac{USP-UVC}{USP} \times 100$$
$$= \frac{20-12}{20} \times 100 = 40\%$$

3- Break-even point sales in units $= \frac{\text{Fixed Cost}}{\text{Selling price per unit} - \text{Variable cost per unit}}$

$$Unit\ Sales\ /_{BEP} = \frac{FC}{USP-UVC}$$
$$= \frac{160,000}{20-12} = 20,000 \text{ Units}$$

4- Break-even point sales in BD $= \frac{\text{Fixed Cost}}{\text{Contribution Margin Percentage}}$

$$Sales\ in\ BD\ /_{BEP} = \frac{FC}{CM\%} =$$
$$\frac{160,000}{40\%} = BD400,000$$

OR

Break-even point in sales BD = Break-even point in sales units × Unit Selling Price

$$Sales\ in\ BD\ /_{BEP} = Unit\ Sales\ /_{BEP} \times USP$$
$$= 20,000 \times 20 = BD400,000$$

2- Teaching Reinforcement Question:
Explain to the students example (4-2-1) pages 108 (Text book.)

3- Formative Assessment.
Ask the student to solve the exercise (4-1) page 115. During the class period by using problem solving strategy.

Study Instructions:
Read text book pages 104-105.

Example 4-2-2:

Refer to example 4-2-1: ABC company sold a computer at BD20 per unit and it had variable cost of BD12 per unit. The total annual fixed cost is BD160,000. If the variable cost per unit increased to BD16.

Required: New break-even after changing variable cost per unit

1) Calculate break-even point sales in units.
2) Calculate break-even point sales in BD.

Answer:

1- Break-even point sales in units = $\frac{\text{Fixed Cost}}{\text{Selling price per unit} - \text{Variable cost per unit}}$

$$Unit\ Sales\ /_{BEP} = \frac{FC}{USP - UVC}$$

$$= \frac{160{,}000}{20-16} = 40{,}000\ \text{Units}$$

2-Break-even point in sales BD = Break-even point in sales units × Unit Selling Price

$$Sales\ in\ BD\ /_{BEP} = Unit\ Sales\ /_{BEP} \times \text{USP}$$

$$= 40{,}000 \times 20 = \text{BD}800{,}000$$

- If the company has an increase in variable cost per unit from BD12 to BD16, it must increase the output from 20,000 units to 40,000 units to get the break-even point.

Teaching with technology

Ask the student to solve the exercise 4-1 page 115.

Teaching Instruction

Explain to the students example. (4-2-2) on pages 108.

➢ **Ask the students to use the following table:**

Chapter Resources			
Resource	**Below Average**	**In Average**	**Over Average**
Teacher's Guide	**Page 115 (EX 4-1.)**	**Page 115 (EX 4-2.)**	**Page 115 (EX4-3.)**
Lesson Resources	**1- Text book 2- Study Guide 3- Digital Educational Lesson 4- YouTube Lesson**	**1- Text book 2- Study Guide 3- Digital Educational Lesson 4- YouTube Lesson**	**1- Text book 2- Study Guide 3- Digital Educational Lesson 4- YouTube Lesson**

Formative assessment:

Ask the students to solve exercises 4-4 page117 (Text Book.)

Text book Exercises

Exercises: page 115

Exercise (4-1):

AL Huda Company sell's office disk for BD30 each and estimated variable costs are expected to be 70% of sales. If the company's fixed costs are BD360,000.

Required:

1- How many office disk must the company sell to break-even?
2- Compute sales revenue for break-even.

Answer:

UVC = 30 × 70% = BD21

$$Unit\ Sales\ /_{BEP} = \frac{FC}{USP - UVC}$$

$$= \frac{360{,}000}{30-21} = 40{,}000 \text{ Units}$$

$$Sales\ in\ BD\ /_{BEP} = Unit\ Sales\ /_{BEP} \times \text{USP}$$

$$= 40000 \times 30 = BD1200{,}000$$

Exercise (4-2):

Some financial data for each of three firms are as follow:

Title	Ahmed's Company	Osama's Company	Abdulla's Company
Selling price per unit	BD100	BD90	BD150
Variable cost per unit	BD80	BD60	BD120
Fixed Cost	BD140,000	BD120,000	BD180,000
Target Operating Income	BD60,000	BD30,000	BD90,000
Unit Sold	8,000 units	7,000 units	9,000 units

Required:

1- What is the break-even point in units and sales revenue for each company?
2- What is the units and sales revenue to get target profit?
3- What is the margin of safety in units?

4- Answer:

Ahmed's Company

1 – $\textbf{Unit Sales}_{/BEP} = \frac{FC}{USP-UVC}$

$$= \frac{140{,}000}{100-80} = 7{,}000 \text{ Units}$$

$$\textbf{Sales in BD}_{/BEP} = \textbf{Unit Sales}_{/BEP} \times \textbf{USP}$$

$$= 7000 \times 100 = \text{BD}700{,}000$$

2 – $\textbf{Unit Sales}_{/Target\ Income} = \frac{FC+TOI}{USP-UVC}$

$$= \frac{140{,}000+ 60{,}000}{100-80} = 10{,}000 \text{ Units}$$

$$\textbf{Sales in BD}_{/Target\ Income} = \textbf{Unit Sales}_{/Target\ Income} \times \textbf{USP}$$

$$= 10{,}000 \times 100 = \text{BD}1000{,}000$$

3- **Margin of Safety = Actual Unit Sales - Break-even point sales in units**

= 8,000 - 7,000 = 1,000 Units

Exercise (4-3):

Zeyad Corporation manufactures a line of computer, the average selling price of its finished product is BD180 per unit. The variable cost per unit is BD110. Zeyad incurs fixed costs of BD630,000.

Required:

1- What is the break-even point in units for the company?
2- What is the sales revenue the firm must achieve to reach the break-even point?

Answer:

$$Unit\ Sales\ /_{BEP} = \frac{FC}{USP-UVC}$$

$$= \frac{630{,}000}{180-110} = 9{,}000 \text{ Units}$$

$$Sales\ in\ BD\ /_{BEP} = Unit\ Sales\ /_{BEP} \times \text{USP}$$

$$= 9000 \times 180 = \text{BD}1620{,}000$$

Exercise (4-4):

Footwear Company manufactures a complete line of men's and women's dress shoes for independent merchants. The average selling price of its finished products is BD85 per pair. The variable cost for this same pair of shoes is BD58. Footwear has fixed costs of BD270,000 per year.

Required:

1- What is the break-even point in " pair of shoes" for the company?
2- What is the sales revenue the firm must achieve to reach the break-even point?
3- If fixed cost increased to BD297,000. What is the new break-even point in pair of shoes and sales revenue?

Answer:

1- $Unit\ Sales\ /_{BEP} = \frac{270{,}000}{85-58} = 10{,}000$ Units

2- $Sales\ in\ BD\ /_{BEP} = 10000 \times 85 = \text{BD}850{,}000$

3- $Unit\ Sales\ /_{BEP} = \frac{297{,}000}{85-58} = 11{,}000$ Units

$Sales\ in\ BD\ /_{BEP} = 11000 \times 85 = \text{BD}935{,}000$

Exercise (4-5):

Fahad Radios manufactures a complete line of radio and communication equipment. The average selling price of its finished products is BD180 per unit. The variable cost for these units is BD126. Fahad Radios incurs fixed costs of BD540,000 per year.

Required:

1- What is the break-even point in radio and communication equipment for the company?
2- What is the sales revenue the firm must achieve to reach the break-even point?
3- What would be the firm's profit or loss at the following units of production sold: 12,000 units? 15,000 units? 20,000 units?

Answer:

1 – $Unit\ Sales\ /_{BEP} = \frac{540{,}000}{180-126}$ = 10,000 Units

2- $Sales\ in\ BD\ /_{BEP}$ = 10000 × 180 = BD1,800,000

3- Profit/Loss = Units Sales × (Selling price per unit – Variable cost per unit) – Fixed Costs

Profit/Loss = [Q× (USP – UVC)] - FC =

Profit/Loss (12000 Units) = [12000× (180 – 126)] - 540000 =BD108000

Profit/Loss (15000 Units) = [15000× (180 – 126)] - 540000 =BD270000

Profit/Loss (20000 Units) = [20000× (180 – 126)] - 540000 =BD540000

Exercise (4-6):

Osama Company expects to earn BD40,000 next year. Sales will be BD400,000, its average product sells for BD20 per unit. The variable cost per unit is BD8.

Required:

1- What are the company's fixed costs expected to be next year?
2- Calculate the company's break-even point in units and sales revenues?

Answer:

1- Profit/Loss = [Q× (USP – UVC)] - FC =

$Q = \frac{400,000}{20}$=20000 units

40000 = [20000× (20 – 8)] – FC

40000 = [20000× (12)] – FC

40000 = 240000 – FC

FC = 240000 -40000 = BD200000

2 – $Unit\ Sales\ /_{BEP}$ = $\frac{200,000}{20-8}$ = 16667 Units

$Sales\ in\ BD\ /_{BEP}$ = 16667 × 20 = BD333,340

Exercise (4-7):

Hamad Company manufactures a full line of lawn furniture. The average selling price of its finished products is BD25 per unit. The associated variable cost for these units is BD15. Fixed costs of BD50,000 per year.

Required:

1- What is the break-even point in units for the company?
2- What is the sales revenue the firm must achieve to reach the break-even point?
3- What would be the firm's profit or loss at the following units of production sold: 4,000 units? 6,000 units? 8,000 units?
4- How many units must be sell to earn an operating income of BD30,000.
5- Compute sale revenue needed to earn an operating income of BD30,000.

Answer:

1 – $Unit\ Sales\ /_{BEP}$ = $\frac{50,000}{25-15}$ = 5,000 Units

2- $Sales\ in\ BD\ /_{BEP}$ = 5000 × 25 = BD125,000

3- Profit/Loss = [Q× (USP – UVC)] - FC =

Profit/Loss (4000 Units) = [4000× (25 – 15)] - 50000 =BD10000 – (Loss)

Profit/Loss (6000 Units) = [6000× (25 – 15)] - 50000 =BD1000

Profit/Loss (8000 Units) = [8000× (25 – 15)] - 50000 =BD30000

4- **Sales in units to earn income** = $\frac{\text{Fixed Cost + Target Operating Income}}{\text{Selling price per unit − Variable cost per unit}}$

$$Unit\ Sales\ /_{Target\ Income} = \frac{FC+TOI}{USP-UVC}$$

$$= \frac{50{,}000+30{,}000}{25-15} = 8{,}000 \text{ Units}$$

5-

$$Sales\ in\ BD\ /_{Target\ Income} = Unit\ Sales\ /_{Target\ Income} \times USP$$

$$= 8000 \times 25 = BD200000$$

Exercise (4-8):

Al Ahmed Company manufactures and sells pens. Currently, 5,000,000 units are sold per year at BD0.5 per unit. Fixed costs are BD900,000 per year. Variable costs are BD0.30 per unit.

Required: consider each case separately

1- a. What is the current annual profit or loss?

b. What is the present break-even in units and revenues?

2- Compute the new profit or loss for each of the following changes:

a. A BD0.04 per unit increase in variable costs.

b. A 10% increase in fixed costs.

c. A 20% decrease in selling price.

3- Compute the new break-even point in units and revenues for each of the following changes:

a. A 20% increase in variable cost per unit.

b. A BD20,000 increase in fixed costs.

Answer:

1- a- Profit/Loss = [Q× (USP – UVC)] - FC

= [5,000,000× (0.5 – 0.3)] – 900,000 = BD100,000

b- $Unit\ Sales\ /_{BEP}$ = $\frac{900{,}000}{0.5-0.3}$ = 4,500,000 Units

$Sales\ in\ BD\ /_{BEP}$ = 4500,000 × 0.5 = BD2,250,000

2- Profit/Loss = [Q× (USP – UVC)] - FC

a- Profit/Loss = [5,000,000× (0.5 – 0.34)] – 900,000= BD100000 – (Loss)

UVC = 0.3 + 0.04 = BD0.34

b- Profit/Loss = [5,000,000× (0.5 – 0.3)] – 990,000= BD10000

FC = 900000 + (900000× 10%) = BD990000

c- Profit/Loss = [5,000,000× (0.4 – 0.30)] – 900,000= BD400000 – (Loss)

USP = 0.5 × (100%-20%) = BD0.4

3- a- $Unit\ Sales\ /_{BEP}$ = $\frac{900{,}000}{0.5-0.36}$ = 6428571 Units

UVC = 0.3 + (0.3× 20%) = BD0.36

$Sales\ in\ BD\ /_{BEP}$ = 6428571× 0.5 = BD3,214,285.5

b- FC = 900000 + 20000 = BD920000

$Unit\ Sales\ /_{BEP}$ = $\frac{920{,}000}{0.5-0.30}$ = 4,600,000 Units

UVC = 0.3 + (0.3× 20%) = BD0.36

$Sales\ in\ BD\ /_{BEP}$ = 4,600,000 × 0.5 = BD2,300,000

Unit 5

Financial Ratio Analysis

4

Learning Objective

- The definition of the financial ratio.
- The calculation of and analyze profitability ratios.
- The benefits of liquidity ratio.
- The calculation of and analyze liquidity ratio.
- The benefits of liquidity ratio.

Unit 5: Financial Ratio Analysis

Lesson Notes

The teacher explains to student:

1- The definition of the financial ratio.

2- The calculation of and analyze profitability ratios.

The Focus

Before the Lesson:
Revision about banking Ratios (Bank211)

During the Lesson:

1- definition of the financial ratio.

2- calculation of and analyze profitability ratios

After the Lesson:

The student able to financial statements analysis by using liquidity ratio

5-2 : Profitability Ratios

Profit is a surplus of revenue over costs and is an absolute money amount. In contrast, **profitability** measures and compares profit relative to business size. In doing so, profitability measures how well or efficiently the business is using its resources to generate profit compared to other businesses.

Although a business may earn a profit it will not be considered profitable if

- Its profit is less than rival business of a similar size selling the same or similar products; and/or
- The owners of the business could earn more profit if they invested their capital in another business venture or even in a bank savings account.

1- The gross profit margin is calculated as follows:

$$\text{Gross profit margin (\%)} = \frac{\text{Gross profit}}{\text{Revenue}} \times 100$$

2- The profit margin is calculated as follows:

$$\text{Profit margin (\%)} = \frac{\text{Profit before tax}}{\text{Revenue}} \times 100$$

3- The Return on capital employed is calculated as follows:

$$\text{ROCE (\%)} = \frac{\text{Profit before tax}}{\text{Capital Employed}} \times 100$$

Capital Employed = Non-current liabilities + Shareholders Funds

OR **= Total assets – Current liabilities**

Fahad Ltd Income statements	BD million	
	Year 1	Years 2
Revenue	**150**	**200**
less cost of sales	90	100
Gross profit	**60**	**100**
less Expenses	15	20
Profit before tax	**45**	**80**
less tax	9	16
Profit after tax	**36**	**64**
Of which		
distributed profit	30	50
retained profit	6	14

Fahad Ltd Statement of financial position	BD million			
	End year 1		End year 2	
Non-current assets		221		295
Cash	10		30	
Inventories	20		15	
Accounts Receivable	14		5	
Current assets	**44**		**50**	
less current liabilities	40		25	
Working capital		4		25
Total assets – current liabilities		**225**		**320**
Financed by				
Non-current liabilities	150		120	
Shareholders' funds	75		200	
Capital employed		**225**		**320**

Answer:

5-2-1: Gross Profit Margin %

	Year 1	Year 2
Gross profit (BD million) ÷ Revenue (BD million)	$\frac{60}{150} \times 100 = 40\%$	$\frac{100}{200} \times 100 = 50\%$

5-2-2: Profit Margin %

	Year 1	Year 2
Profit before tax (million BD) ÷ Revenue (million BD)	$\frac{45}{150} \times 100 = 30\%$	$\frac{80}{200} \times 100 = 40\%$

5-2-3: Return on Capital Employed % (ROCE)

	Year 1	Year 2
Profit before tax (million BD) ÷ Capital employed (million BD)	$\frac{45}{225} \times 100 = 20\%$	$\frac{80}{320} \times 100 = 25\%$

2- Teaching Reinforcement Question:
Explain to the students example (5-2-1) & (5-2-2) pages 115-128 (Text book.)

3- Formative Assessment.
Ask the student to solve the activity (5-2-1) page 130. During the class period by using problem solving strategy.

Activity 5-2-1:

- MTA Wholesalesalers is a limited company operating in Bangladesh. It has been operating successfully for over 11 years. Below are some key results from its financial statements for the last two years.

MTA wholesalers Pvt Ltd Summarized results from financial statements		
	Year 10 BD (000)	**Year 11 BD (000)**
Revenue	400	420
Cost of sales	240	252
Gross profit	?	?
Overheads	130	147
Profit for the year	?	?
Capital employed	200	210

Required:

From the information presented calculate each year the company's:

- Gross Profit %.
- Gross Profit Margin %.
- ROCE%.

Teaching with technology

Ask the student to solve the exercise 5-1 page 138.

Teaching Instruction

Explain to the students example.
To read pages 122-123.

Chapter Resources			
Resource	**Below Average**	**In Average**	**Over Average**
Teacher's Guide	**Page 138 (EX5-1.)**	**Page 138 (EX5-2.)**	**Page 139 (EX5-3 From 1 to 5.)**
Lesson Resources	**1- Text book 2- Study Guide 3- Digital Educational Lesson 4- YouTube Lesson**	**1- Text book 2- Study Guide 3- Digital Educational Lesson 4- YouTube Lesson**	**1- Text book 2- Study Guide 3- Digital Educational Lesson 4- YouTube Lesson**

Formative assessment:

Ask the students to solve exercises 5-1 page138 (Text Book.)

Text book Exercises

Activity 5-2-1:

- MTA Wholesalesalers is a limited company operating in Bangladesh. It has been operating successfully for over 11 years. Below are some key results from its financial statements for the last two years.

MTA wholesalers Pvt Ltd Summarized results from financial statements		
	Year 10 BD (000)	**Year 11 BD (000)**
Revenue	400	420
Cost of sales	240	252
Gross profit	160	168
Overheads	130	147
Profit for the year	30	21
Capital employed	200	210

Required:

From the information presented calculate each year the company's:

- Gross Profit.
- Gross Profit Margin %.
- ROCE%.

Answer:

- **Gross Profit = Revenue - Cost of sales**

  Gross profit year 1: 400-240= BD160

  Gross profit year 2: 420-252=BD168

- **Gross profit margin (%) = $\frac{\text{Gross profit}}{\text{Revenue}} \times 100$**

  Gross profit margin year1: $\frac{160}{400} \times 100 = 40\%$

  Gross profit margin year2: $\frac{168}{420} \times 100 = 40\%$

➢ **ROCE (%) = $\frac{\text{Profit before tax}}{\text{Capital Employed}} \times 100$**

Profit before tax year1 = Gross profit – Expenses
= 160 – 130 =30
Profit before tax year2 = Gross profit – Expenses
= 168 – 147 =21

ROCE year1: $\frac{30}{200}\times100=15\%$
ROCE year2: $\frac{21}{210}\times100=10\%$

Exercises:

Exercise (5-1):

The following results were extracted from statements of financial position prepared for Abdulla's Company at the end of years 2018 and 2019.

Abdulla's Company Summarized results from financial statements at 31 December		
	Year 2018 BD (000)	**Year 2019 BD (000)**
Cash	50	80
Inventories	40	60
Account Receivable	30	20
Total Current Assets	**120**	**160**
Account payable	15	20
Bank Overdraft	10	30
Total Current Liabilities	**25**	**50**
Total Assets	**300**	**350**

Required:

From the information presented calculate for the company's each year its:

1- Working Capital.
2- Capital Employed.

3- Current Ratio.

4- Acid Test Ratio (Quick Ratio)

Answer:

(1) Working capital year1 = Current Assets – Current Liabilities

= 120 – 25 =95

Working capital year2: 160 – 50 = 110

(2) Capital employed year1 = Total assets – current liabilities

= 300 – 25 = 275

Capital employed year 1 = 350– 50 = 300

(3) **Current Ratio year 1 =** $\frac{current\ assets}{current\ liabilities} = \frac{120}{25}$ = 4.8 Times

Current Ratio year 2 = $\frac{160}{50}$ = 3.2 Times

(4) **Quick Ratio year 1 =** $\frac{current\ assets - inventory}{current\ liabilities} = \frac{120-40}{25}$ = 3.2 Times

Quick Ratio year 2 = $\frac{160-60}{50}$ = 2 Times

Exercise (5-2):

The following as the income statements for Salman Company for years 2019 and 2020.

Salman Company Income Statements	BD million	
	Year 2019	Year 2020
Revenue	500	900
less cost of sales	300	450
Gross profit	**200**	**450**
less expenses	40	225
Profit before tax	**160**	**225**
less tax	60	80
Profit after tax	**100**	**145**

Required:

From the information presented, calculate for the company's each year its:

1- Gross Profit Margin %.

2- Profit Margin %.

3- Return on Capital Employed (%).

If Capital Employed:

2019 = 320 & 2020 = 337.5

Answer:

1- Gross profit margin (%) = $\frac{\text{Gross profit}}{\text{Revenue}} \times 100$

Gross profit margin year 1 = $\frac{200}{500} \times 100 = 40\%$

Gross profit margin year 2 = $\frac{450}{900} \times 100 = 50\%$

2- Profit margin (%) = $\frac{\text{Profit before tax}}{\text{Revenue}} \times 100$

Profit margin year 1 = $\frac{160}{500} \times 100 = 32\%$

Profit margin year 2 = $\frac{225}{900} \times 100 = 25\%$

3- ROCE%= $\frac{profit\ before\ tax}{capital\ employed} \times 100$

ROCE 2019= $\frac{160}{320} \times 100 = 22.86\%$

ROCE 2020= $\frac{225}{337.5} \times 100 = 21.53\%$

Exercise (5-3):

The balance sheet and income statement for MRG Company are as

Balance Sheet	BD 000
Cash	500
Account Receivable	2,000
Inventories	1,000
Current Assets	**3,500**
Fixed Assets	4,500
Total Assets	**8,000**
Current Liabilities	2,000
Long-term debt	2,000
Owners' Equity	4,000
Total Liabilities and equity	**8,000**

Income Statement	BD 000
Net Sales (Revenues)	8,000
- Cost of Goods Sold	3,200
Gross Profit	**4,800**
- Operating Expenses	800
Operating Income	**4,000**
Interest Expenses	1,000
Profits before taxes	**3,000**
Tax (5%)	150
Net Income	**2,850**

Required:

Calculate the following ratios:

1- Gross Profit Margin %.
2- Profit Margin %.
3- Return on Capital Employed (%).
4- Working Capital.
5- Capital Employed.
6- Current Ratio.
7- Acid Test Ratio (Quick Ratio).

Answer:

1- Gross profit margin (%) = $\frac{\textbf{Gross profit}}{\textbf{Revenue}} \times \textbf{100} = \frac{\textbf{4800}}{\textbf{8000}} \times 100 = 60\%$

2- Profit margin (%) = $\frac{\textbf{Profit before tax}}{\textbf{Revenue}} \times \textbf{100} = \frac{3000}{8000} \times 100 = 37.5\%$

3- ROCE%= $\frac{\textbf{\textit{profit before tax}}}{\textbf{\textit{capital employed}}} \times \textbf{100}$

Capital employed = Total assets – Current liabilities

= 8000-2000 = BD6000

ROCE = $\frac{3000}{6000} \times 100 = 50\%$

4- Working Capital = 3500 – 2000 = BD1500

5- Capital employed = Total assets – Current liabilities

= 8000-2000= BD6000

6- Current Ratio = $\frac{3500}{2000}$ = 1.75 times

7- Acid Test Ratio (Quick Ratio). = $\frac{3500-1000}{2000}$ = 1.25 times

Exercise (5-4):

The annual sales for Sara Company were BD4.5 million last year, and the value of inventories was BD 120,000. The firm's end-of-year balance sheet and income statement appeared as follows:

Balance Sheet	BD 000
Current Assets	500,000
Net Fixed assets	1,500,000
Total assets	2,000,000
Current liabilities	400,000
Owner's Equity	1,600,000
Total liabilities and Equity	2,000,000

Income Statement	BD
Sales (Revenues)	4,500,000
- Cost of Goods Sold	3,500,000
Gross Profit	**1,000,000**
- Operating Expenses	(500,000)
Operating Income	**500,000**
Interest Expenses	100,000
Profits before taxes	**400,000**
Tax (5%)	20,000
Net Income	**380,000**

Required:

Calculate the following ratios:

1- Gross Profit Margin %.
2- Profit Margin %.
3- Return on Capital Employed (%).
4- Working Capital.
5- Capital Employed.
6- Current Ratio.
7- Acid Test Ratio (Quick Ratio).

Answer:

1- Gross Profit Margin % = $\frac{1000000}{4500000} \times 100 = 22.22\%$

2- Profit Margin %. = $\frac{400000}{4500000} \times 100 = 8.89\%$

3- Capital Employed = 2000000 – 400000 = BD1600000

ROCE= $\frac{400000}{1600000}$×100 = 25%

4- Working capital = Current Assets – Current Liabilities

= 500000 – 400000 = 100000

5- Capital Employed = 2000000 – 400000 = 1600000

6- Current Ratio = $\frac{500000}{400000}$ = 1.25 Times

7- Acid Test Ratio (Quick Ratio). = $\frac{500000-120000}{400000}$ = 0.95 Times

Exercise (5-5):

Balance Sheet	BD 000
Cash	1,000
Account Receivable	1,500
Inventories	1,000
Current Assets	**3,500**
Fixed Assets	4,500
Total Assets	**8,000**
Account Payable	1,000
Accrued Expenses	800
Total Current Liabilities	**1,800**
Long-term debt	2,100
Owners' Equity	4,100
Total Liabilities and equity	**8,000**

Income Statement	BD 000
Net Sales (Revenues)	8,000
- Cost of Goods Sold	3,200
Gross Profit	**4,800**
- Operating Expenses	800
Operating Income	**4,000**
Interest Expenses	1,000
Profits before taxes	**3,000**
Tax (5%)	150
Net Income	**2,850**

Required:

Calculate the following ratios:

1- Gross Profit Margin %.
2- Profit Margin %.
3- Return on Capital Employed (%).
4- Working Capital.
5- Capital Employed.
6- Current Ratio.
7- Acid Test Ratio (Quick Ratio).

Answer:

1- **Gross Profit Margin % =** $\frac{4800}{8000} \times 100 = 60\%$

2- **Profit Margin %.** = $\frac{3000}{8000} \times 100 = 37.5\%$

3- **Capital employed =** 8000 – 1800 = 6200

ROCE = $\frac{3000}{6200} \times 100 = 48.39\%$

4- **Working capital =** 3500 – 1800 = 1700

5- **Capital employed =** 8000 – 1800 = 6200

6- **Current Ratio** $= \frac{3500}{1800} =$ 1.94 Times

7- **Acid Test Ratio (Quick Ratio).** $\frac{3500-1000}{1800}$ = 1.39 Times

Exercise (5-6):

Income Statement	2017 BD000	2018 BD000	2019 BD000	2020 BD000
Sales (Revenues)	3479	3644	3225	2,900
- Cost of Goods Sold	2109	2255	1997	1,746
Gross Profit	**1370**	**1389**	**1228**	**1,154**
- Operating Expenses	1105	1113	1088	1,015
Operating Income	**265**	**276**	**140**	**139**
Interest Expenses	35	56	20	29
Profits before taxes	**230**	**220**	**120**	**110**
Tax (5%)	11.5	11	6	5.5
Net Income	**218.5**	**209**	**114**	**104.5**

Balance Sheet	2017 BD000	2018 BD000	2019 BD000	2020 BD000
Cash	201	327	339	309
Account Receivable	507	591	562	518
Inventories	635	545	564	315
Current Assets	**1343**	**1463**	**1465**	**1362**
Fixed Assets	313	323	291	378
Total Assets	**1656**	**1786**	**1756**	**1740**
Current Liabilities	432	517	557	612
Long-term debt	254	888	692	587
Owners' Equity	970	381	507	541
Total Liabilities and equity	**1656**	**1786**	**1756**	**1740**

Required:

Calculate the following ratios for each year:

1- Gross Profit Margin %.
2- Profit Margin %.
3- Return on Capital Employed (%).
4- Working Capital.
5- Capital Employed.
6- Current Ratio.
7- Acid Test Ratio (Quick Ratio).

Answer:

(1) Gross Profit Margin %

2017= $\frac{1370}{3479}$×100 = 39.38%

2018 = $\frac{1389}{3644}$×100 = 38.12%

2019 = $\frac{1228}{3225}$×100 = 38.08%

2020 = $\frac{1154}{2900}$×100 = 39.79%

(2) Profit Margin %

2017= $\frac{230}{3479}$×100 = 6.61%

2018 = $\frac{220}{3644}$×100 = 6.04%

2019 = $\frac{120}{3225}$×100 = 3.72%

2020 = $\frac{110}{2900}$×100 = 3.79%

(3) Capital employed = Total Assets – Current Liabilities

2017 = 1656 – 432 = 1224
2018 = 1786 – 517 = 1269
2019 = 1756 – 557 = 1199
2020 = 1740 – 612 = 1128

ROCE 2017= $\frac{230}{1224}$×100 = 18.79%

2018 = $\frac{220}{1269}$×100 = 17.34%

2019 = $\frac{120}{1199}$×100 = 10.01%

2020 = $\frac{110}{1128}$×100 = 9.75%

(4) Working Capital= Current Assets - Current Liabilities

2017 = 1343 – 432 = 911

2018 = 1463 – 517 = 946

2019= 1465 – 557 = 908

2020 = 1362 – 612 = 750

(5) Capital Employed 2017 = 1656 – 432 = 1224

2018 = 1786 – 517 = 1269

2019 = 1756 – 557 = 1199

2020 = 1740 – 612 = 1128

(6) Current Ratio = $\frac{\textit{Current Assets}}{\textit{Current Liabilities}}$

2017 = $\frac{1343}{432}$ = 3.11 Times

2018 = $\frac{1463}{517}$ = 2.83 Times

2019 = $\frac{1465}{557}$ = 2.63 Times

2020 = $\frac{1362}{612}$ = 2.23 Times

(7) Quick Ratio = $\frac{\textit{Current Assets}-\textit{Inventories}}{\textit{Currnt Liabilities}}$

2017 = $\frac{1343-635}{432}$ = 1.64 Times

2018 = $\frac{1463-545}{517}$ = 1.78 Times

2019 = $\frac{1465-564}{557}$ = 1.62 Times

2020 = $\frac{1362-315}{612}$ = 1.71 Times